C. A. Petri Milano, Apr. 1989

1

CONCURRENCY THEORY

and

COMBINED AXIOMATICS FOR
- CONCURRENCY
- CAUSALITY
- POSSIBILITY

I

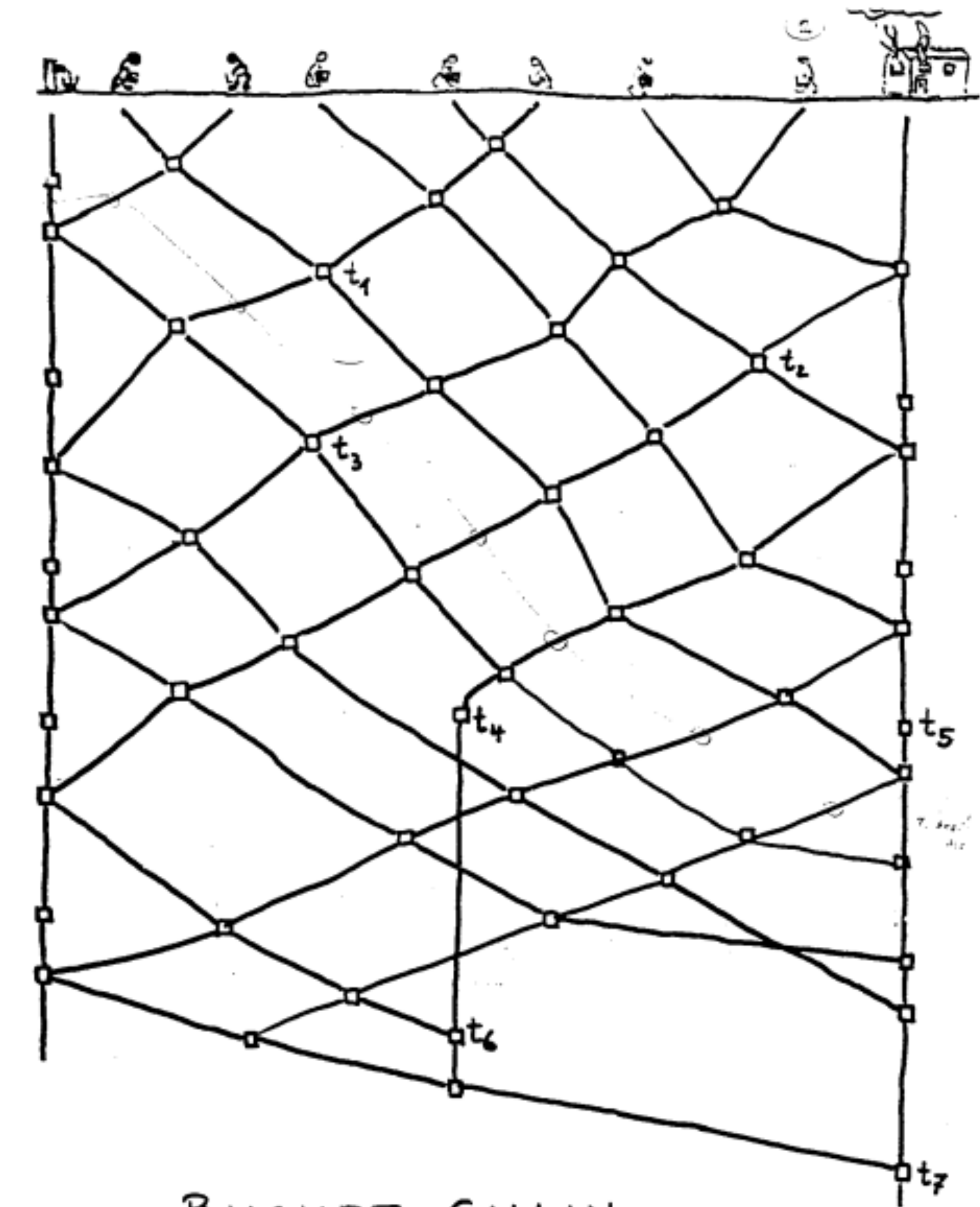
(1.) Introduction by examples:
   - Bucket chain
   - Secure transmission Lines

(2.) Basic assumptions on concurrency:
   - Simplicity
   - Coherence
   - K-density

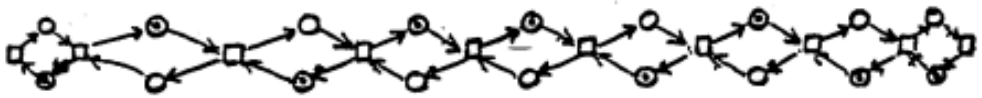
(3.) Complete Orders of Signals
   - $\Sigma := (H; al, bi, co)$
   - Axioms, Concepts
   - Interpretation
   - Perfect nets

(4.) Mathematics of Signal Combinatorics:
   - Kens
   - Cycloids
   - Splines

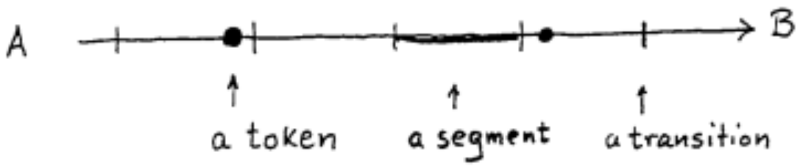
(5.) Relation to Physics and Pragmatics



BUCKET CHAIN:

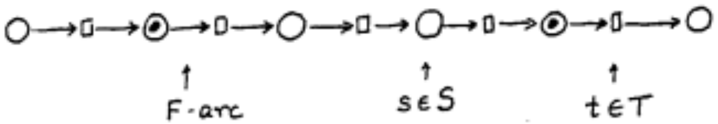


Safety and Security. .1



<u>a marked graph</u>

(all segments directed towards B!)



<u>a marked net</u> $(S, T; F, C)$

$S \cup T \neq \emptyset$

$S \cap T = \emptyset$

$F \subseteq S \times T \cup T \times S$

$\operatorname{dom} F \cup \operatorname{ran} F = S \cup T$

$C \subseteq \wp S$ a marking class

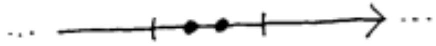
$X := S \cup T$ the net elements.

$x \in X$: $\quad {}^{\bullet}x := \{y \mid y F x\}$

$x^{\bullet} := \{y \mid x F y\}$

$\dot{x} := \{x\}$

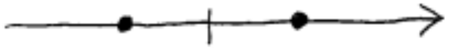
Safety and Security .2



an <u>accident</u>

(two trains in the space = segment which can contain one [undamaged] train only)

(cf. two messages ... etc.)



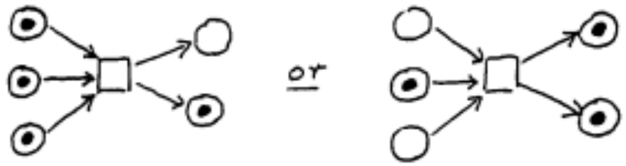
a <u>contact</u> situation

(the situation immediately before an accident)

Definition: $t \in T, c \in C$:

$\text{Contact}(t, c) :\Leftrightarrow {}^{\bullet}t \subseteq c \wedge t^{\bullet} \cap c \neq \emptyset$

<u>or</u> $t^{\bullet} \subseteq c \wedge {}^{\bullet}t \cap c \neq \emptyset$

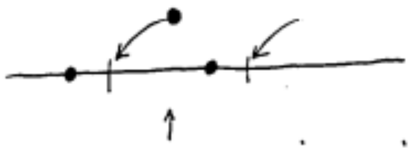


Note: The def. is <u>F-reversal-invariant</u>

PROBLEM: HOW TO AVOID CONTACT?

Safety and Security .3

How to avoid contact?



a warning signal
„inhibits the transition"

Not a safe solution!
(Signals, like trains, may be delayed)

a permission signal

A safe solution:
Proof: Every S-element (segment) lies on a basic circuit

which contains one token (train or permit) at the time of construction; therefore in all later situations.

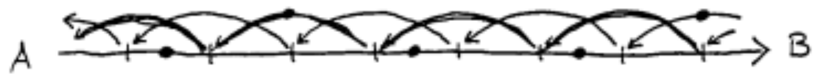
(Def. Safe net : no contact in C)
Safety criterion : $\Lambda s \in S$ on a basic set.

Safety and Security . 4

The tokens are now safe. Are the trains safe?

No.

1. Trains have nonzero length:



∴ At least two segments must be reserved for each train

2. Trains have nonzero mass:

∴ At least three segments must be reserved for each train.

The trains are now sufficiently separated.

Are they safe?

No.

The permit signals have
(zero mass if electromagnetic, but:)

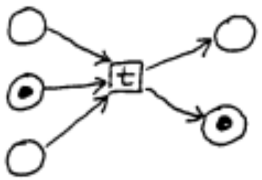
NONZERO LENGTH

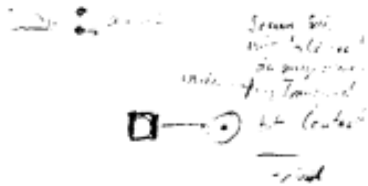
TASK: CONSTRUCT., DEFIN., CRITERION!

DEF.: <u>Transjunction</u> $(t,c) :\Leftrightarrow$

$$^{\bullet}t \cap c \neq \emptyset \ \wedge \ t^{\bullet} \cap c \neq \emptyset$$

Examples:





(<u>Transjunction</u> is the con<u>junction</u> of conditions across a <u>transition</u>)

DEF.: <u>Secure</u> $(S, T; F, C) :\Leftrightarrow$

$$\bigwedge t \in T, c \in C : \neg \text{Transjunction}(t,c)$$
$$\underline{\text{and}} \ \neg \text{Contact}(t,c) \ .$$

where $(S,T;F)$ is a net or digraph and C the <u>full</u> marking class.

Note : The Def's are F-reversal-invariant.

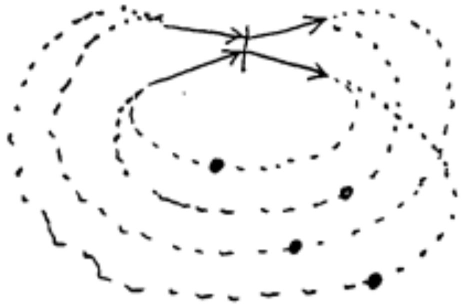
<u>PROBLEM</u> : Can every net be made secure ?

If so, how ?

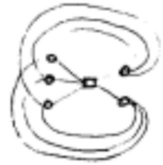
<u>Security Criterion</u>:

1. For marked digraphs:

Every <u>pair</u> of successive arcs must lie on a <u>basic</u> circuit :

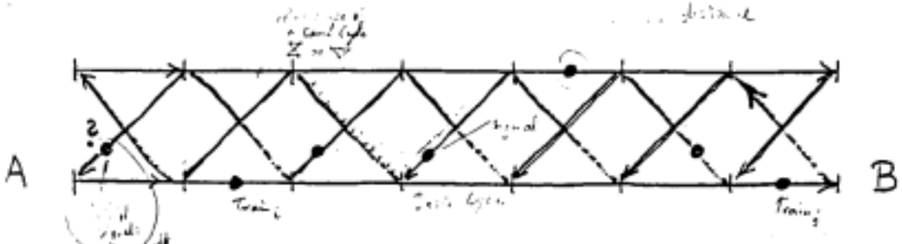


2. For (elementary) nets:



Every transition with
m inputs (preconditions) and
n outputs (postconditions)
must lie on $m \cdot n$ <u>basic domains</u>
(signal domains without concurrency)
covering $^{\bullet}t^{\bullet}$.

Example: Secure transmission line for massless (e.m.) signals:

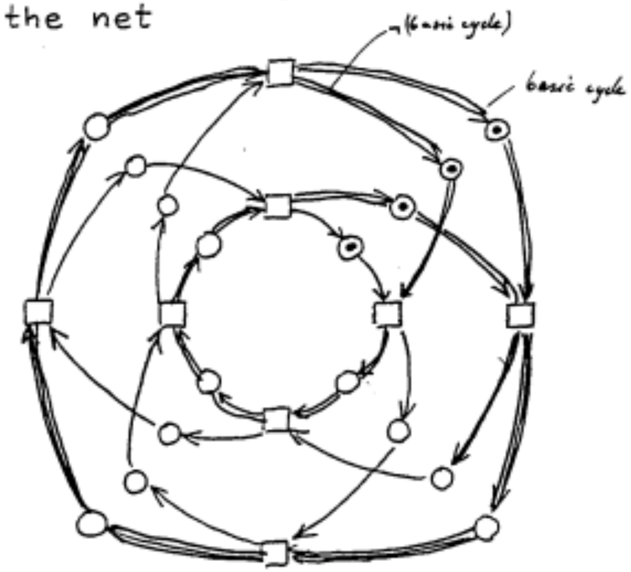


TASK: Characterize <u>all</u> secure digraphs!

CYCLOIDS. I.

Domains and Effects have common structural properties which ~~are~~ may be of mathematical interest, quite apart from physics or signal systems. Some concurrency structures are, mathematically, equivalent to „cycloids", as we may call those structures, similar to matroids and graphoids, but being simple undirected GRAPHS with say 24 interesting properties which have practical relevance, when seen in conjunction.

The smallest cycloid may be <u>denoted</u> by the net



and <u>is</u> the graph with vertices $S \cup T$, and edges the relation bi of this net.
(184 edges!) (Li)

CYCLOIDS, II

EXERCISE: DRAW THE CONCURRENCY GRAPH OF THIS NET. (IT HAS 24 VERTICES AND ONLY 92 EDGES)
HINT: SPREAD THE 16 S-ELEMENTS OVER A FOUR-DIMENSIONAL CUBE AND DRAW FIRST THE EDGES co|S!

The graph $(X; li)$ of this (and all) cycloid has the following remarkable (in conjunction!) properties:

1. Definable by 1 symmetric relation $(li;\ X := \text{field}\ li)$
2. Definable in terms of 1 set $\mathbb{Q}$ of separation quads.
3. Combinatorial (= nowhere dense)
4. Coherent $(Li^* = \overline{Li}^*)$
5. Kdense : $\text{Kens}(Li) \not\otimes \text{Kens}(\overline{Li})$
6. Net, of rank $(1 \cdot \overline{2})$ or $(\overline{2 \cdot 1})$
7. 1 safe-and-live marking class, computable from li!
8. secure (resp. Markovian)
9. simple = irreducible : $\widetilde{li} = \widetilde{\widetilde{li}} = id$
10. T_0 - topology
11. directable in precisely 2 consistent ways
12. coverable by 1 Eulerian cycle
13. coverable by 1 Hamiltonian cycle
14. strongly connected when directed
15. regular colourings exist
16. finite
17. unending when directed
18. possesses primitive topologies (2)
19. possesses elementary topologies (2)

CYCLOIDS, III

20: possesses G-Dedekind-cuts „everywhere"

21: is G-Dedekind-continuous (no jumps, no gaps)

22: possesses plain covering by basic Lines

23: possesses plain covering by nonbacis Lines (!)

24: possesses two separation-quad sets: $\mathbb{Q}$, and $\mathbb{Q}'$ computable from $\mathbb{Q}$.

Cycloids with additional number-theoretic properties can be systematically constructed (Yuan, 1989)

In Computing, cycloids resemble secure token rings.

In Physics, they may be conjectured to represent eigenfunctions of the Schrödinger equation.

A small subset of properties 1-24 above is sufficient to define the concept „CYCLOID". Lacking (at this moment) the tools of computer algebra, we give an untested (<u>provisional</u>)

<u>DEFINITION</u>: CYCLOID $(X; li) :\Longleftrightarrow$ (Vco:)

CY1: $li \cup co \cup id = X \times X$

CY2: $li \cap co = co \cap id = id \cap li = \phi$

CY3: $li^{-1} = li$

CY4: $\widetilde{li} = \widetilde{co} = id$

CY5: $li^* = co^*$

CY6: Kens (li) $\not\#$ Kens (co)

CY7: Kens(li) # Kens(li) $\wedge$ Kens(co) # Kens(co)

CY8: $im^* = \overline{im}^* \wedge im^2 \subseteq \overline{im}$ where $im := L \cup L^{-1}$

CY9: $L \cup L^{-1} = im \wedge L^2 = \phi$ where $L := li - li \cdot co$

CY10: Within each im [x]: $id = co^2 \subseteq li^2 \subseteq \overline{li} \wedge co^2 \subseteq \underline{co}$

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APPENDIX:

SOME REMARKS ON

- CONTINUITY
- „OPERATIONAL" TOPOLOGY
- MATH. PRAGMATICS.

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