PLACE FOLDINGS OF REGULAR CYCLOIDS FOR MODELLING STOP TOLERANCE

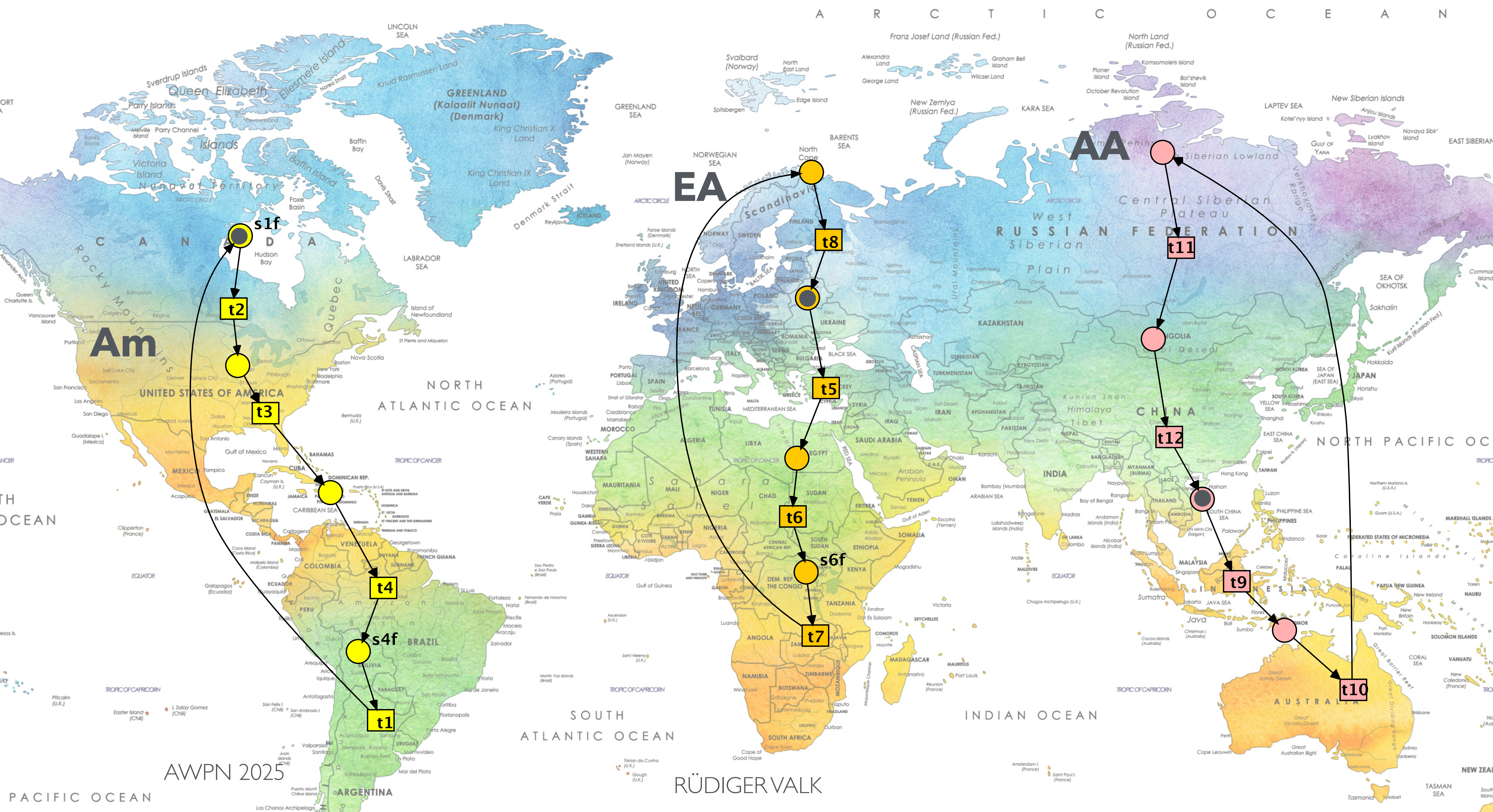
RÜDIGER VALK

UNIVERSITY OF HAMBURG

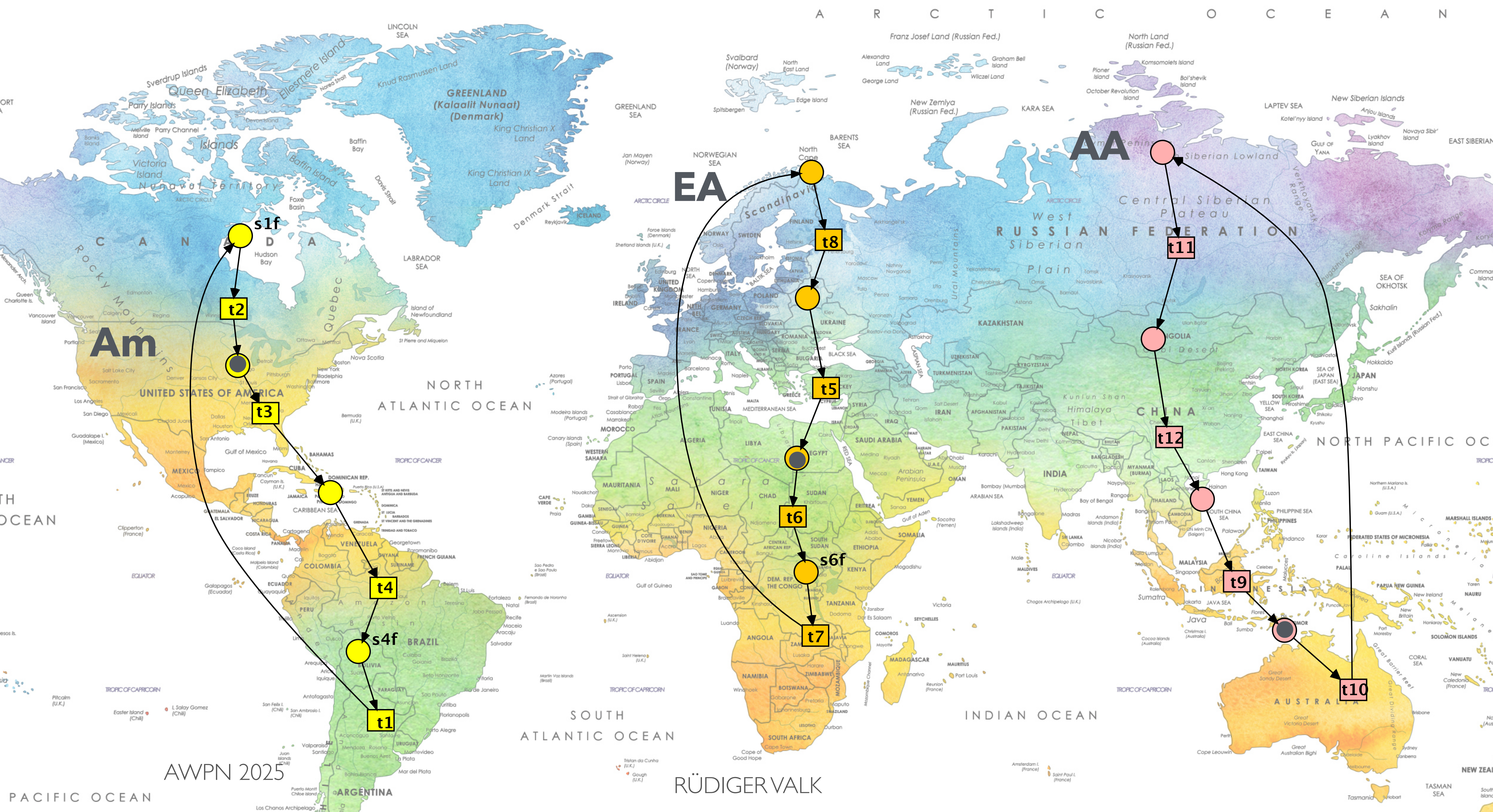
AWPN 2025

AUGSBURG, SEPTEMBER 18/19, 2025

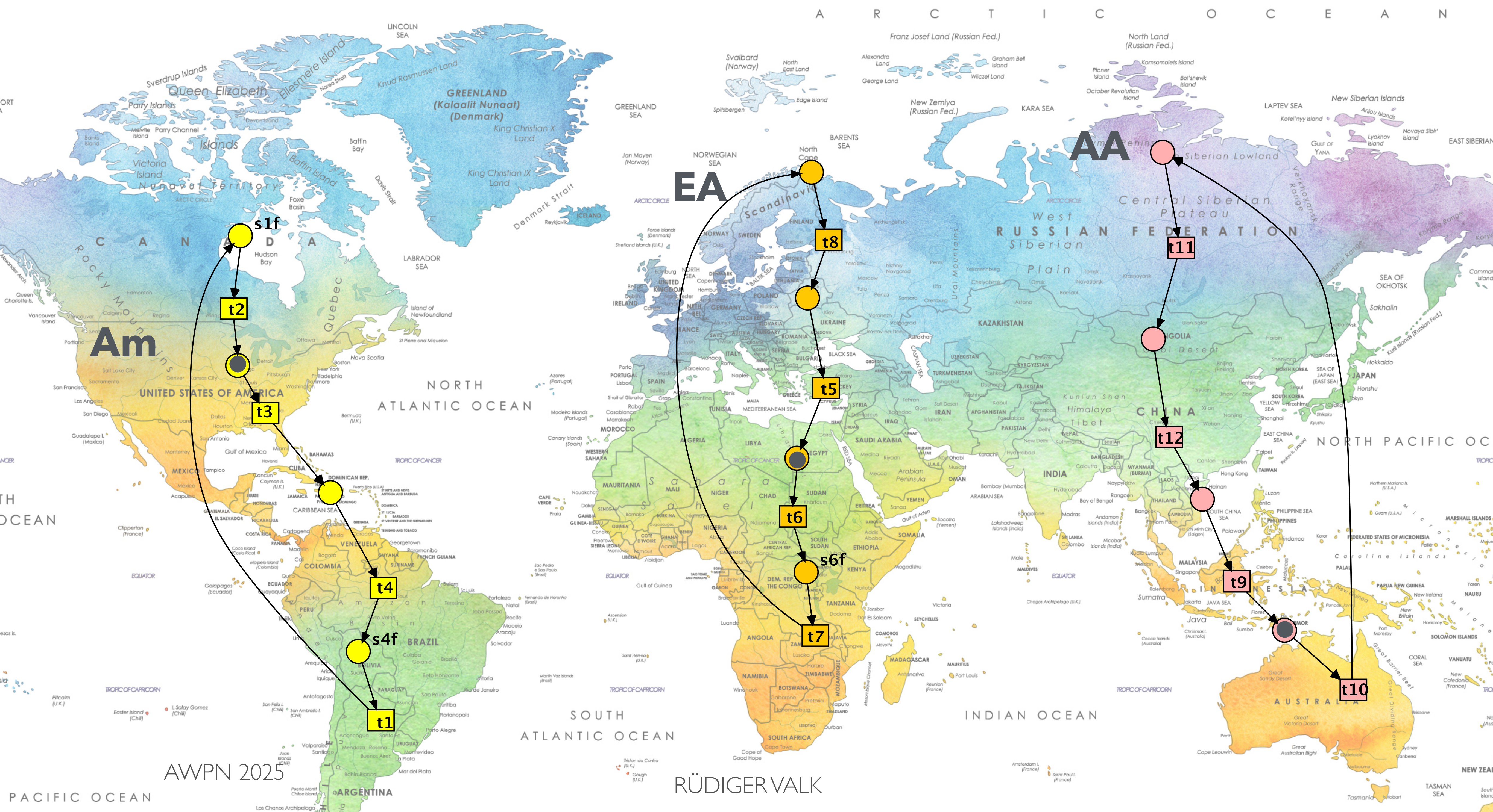
system of sequential processes



system of sequential processes



system of sequential processes
What common behaviour is specified ?



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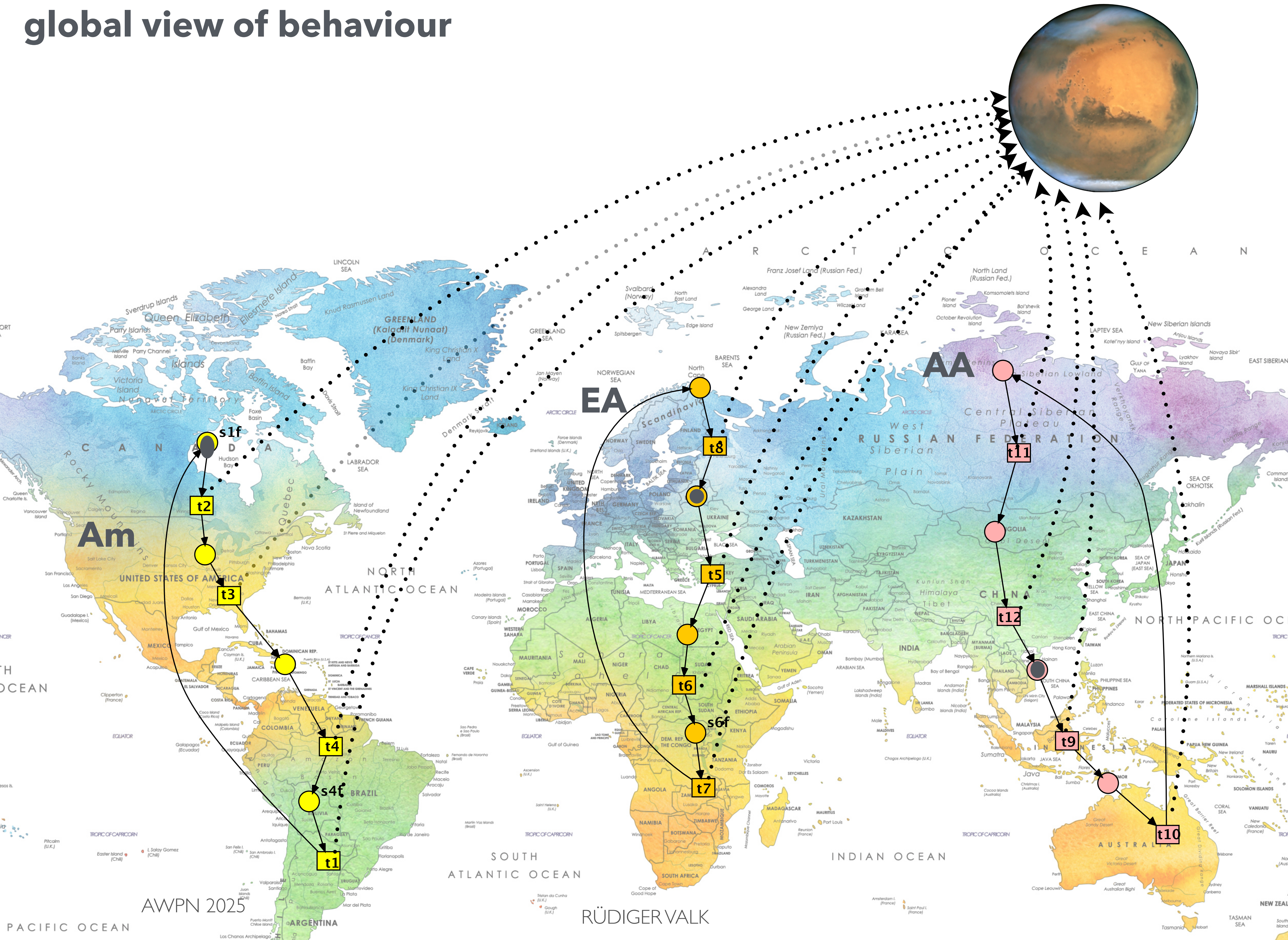
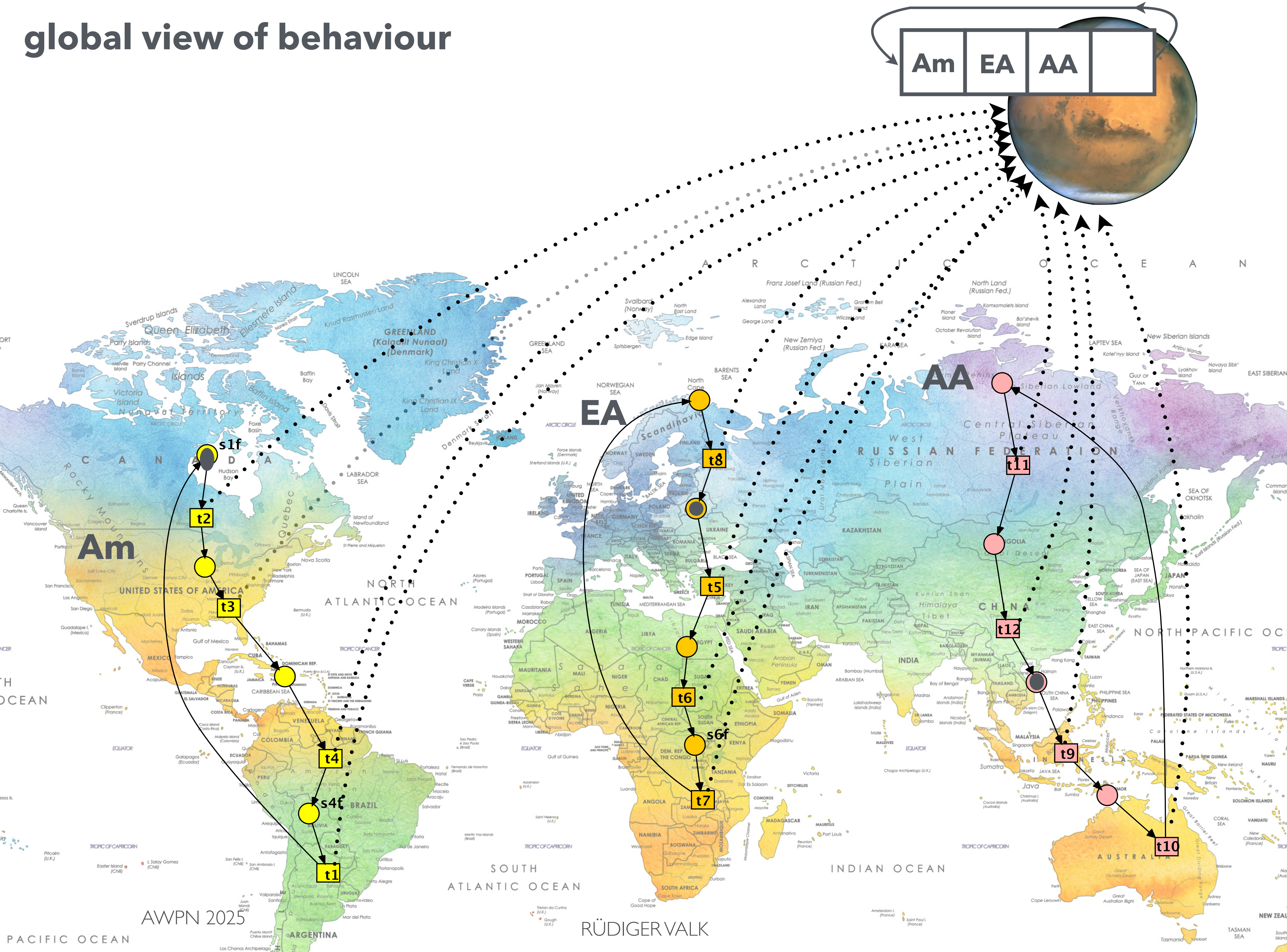
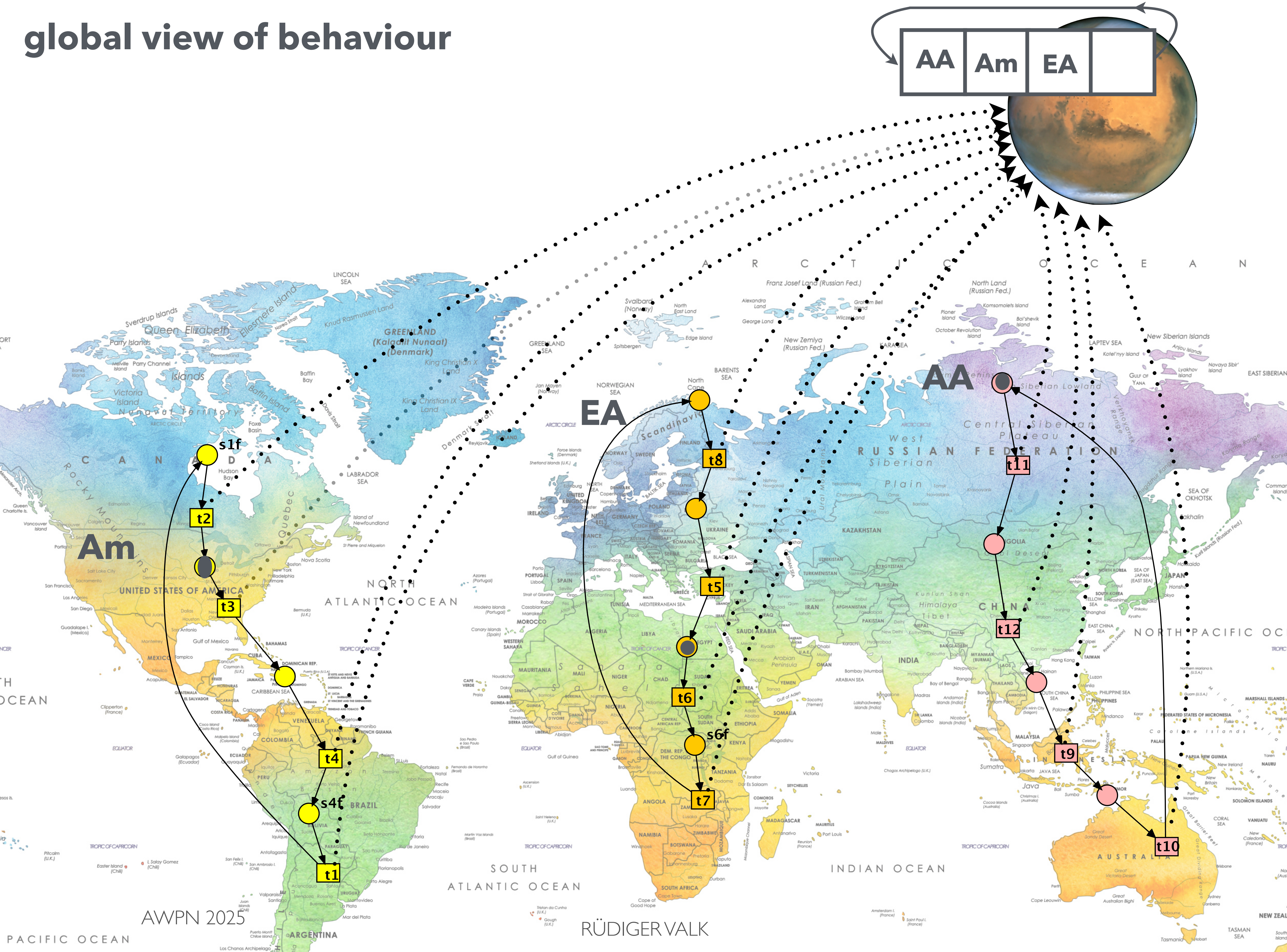
global view of behaviour



global view of behaviour



global view of behaviour



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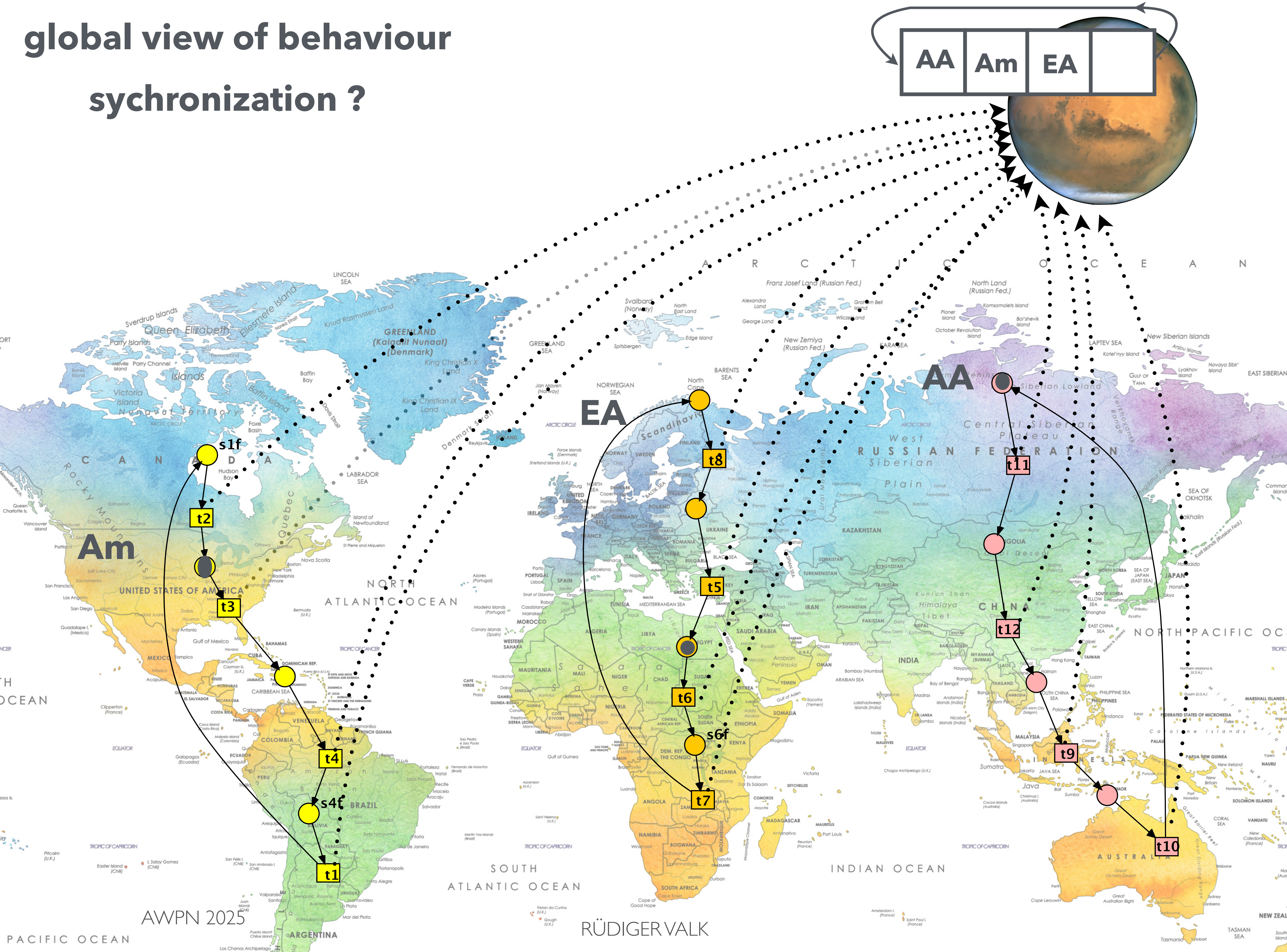
global view of behaviour sychronization ?



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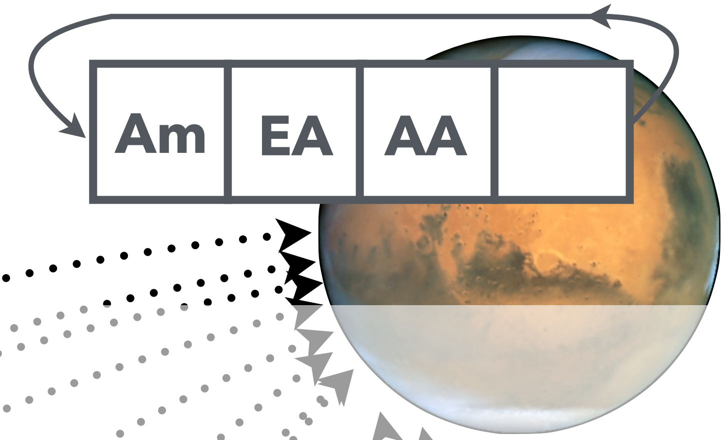
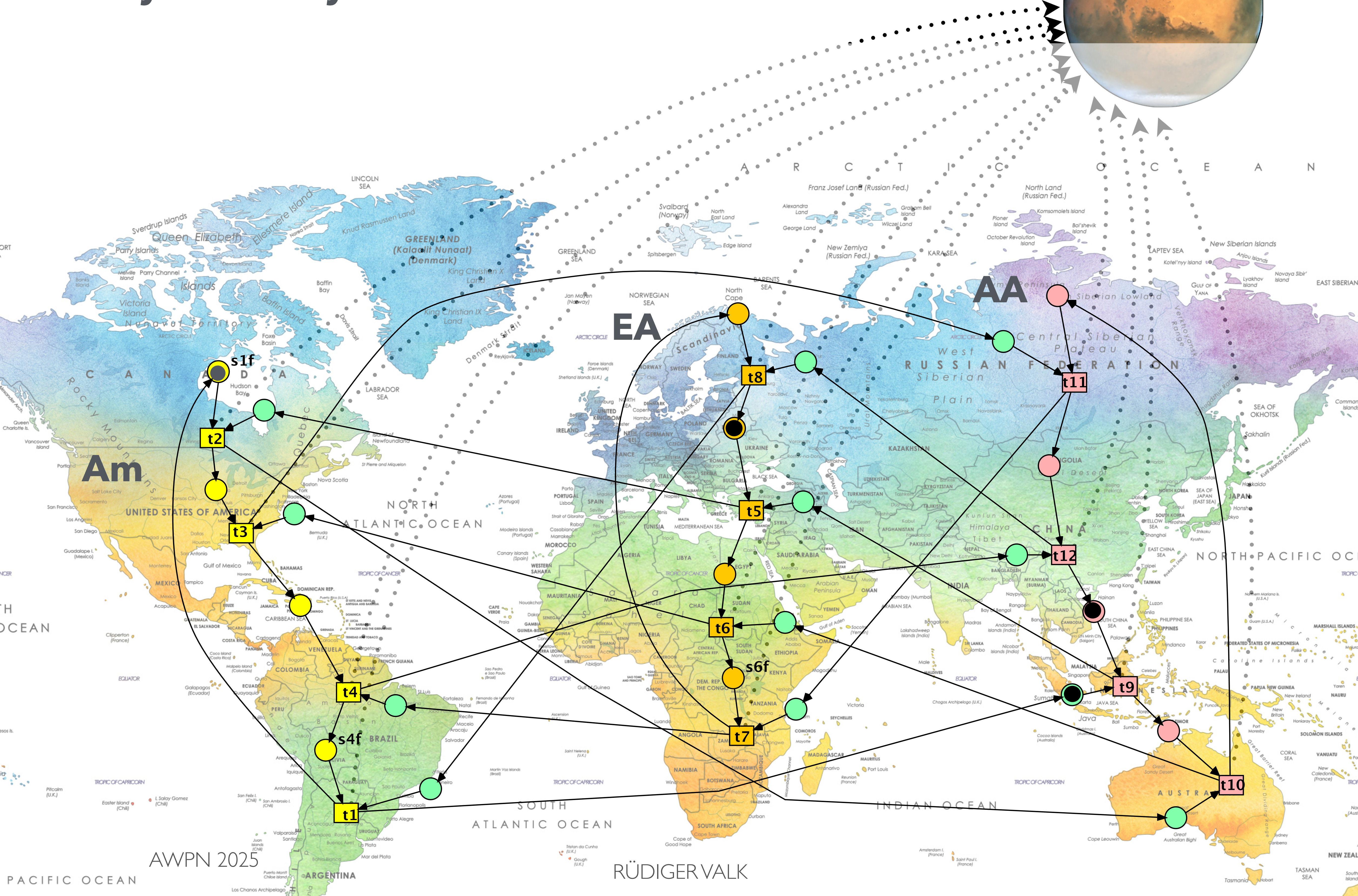
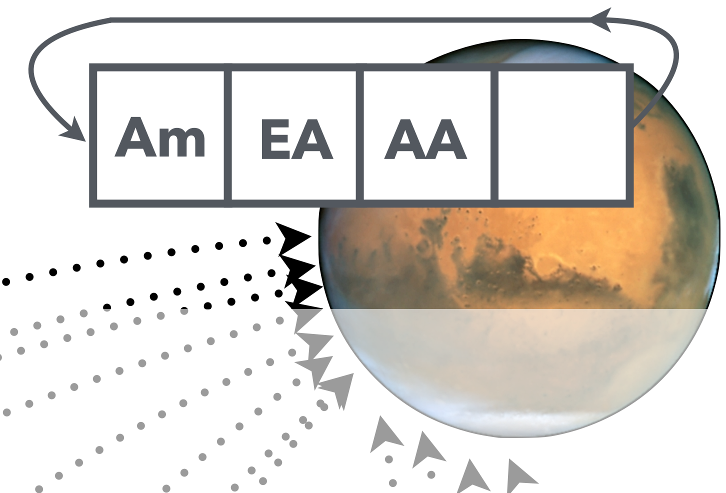
sychronization ?
by Petri's "cycloids"

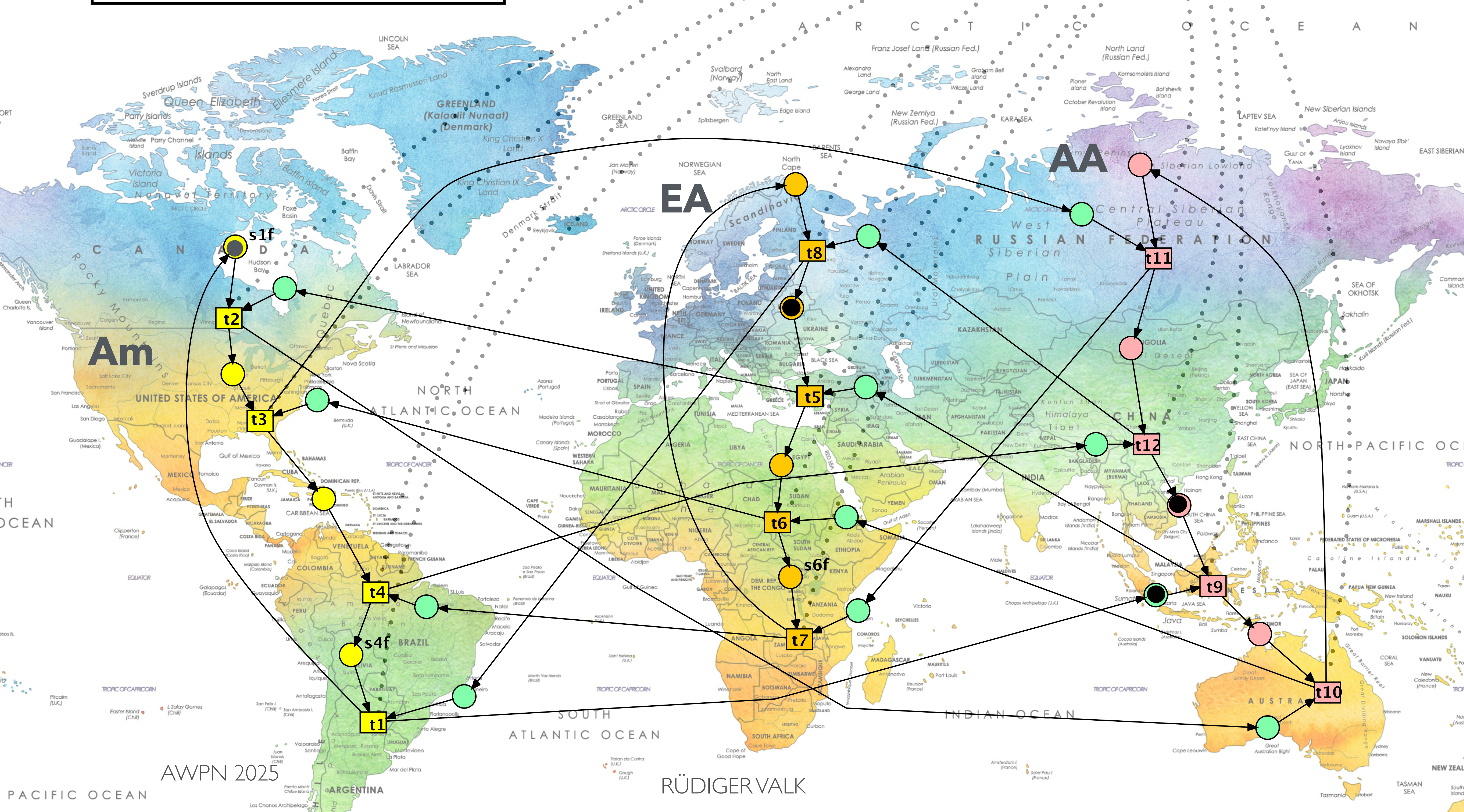




sychronization ?
by Petri's "cycloids"

Nets, time and space
C.A. Petri
Theoretical Computer Science 153 (1996) 3–48





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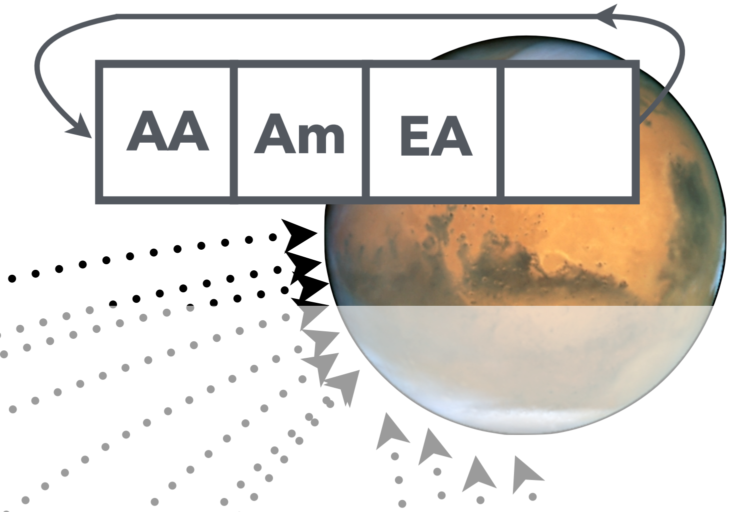

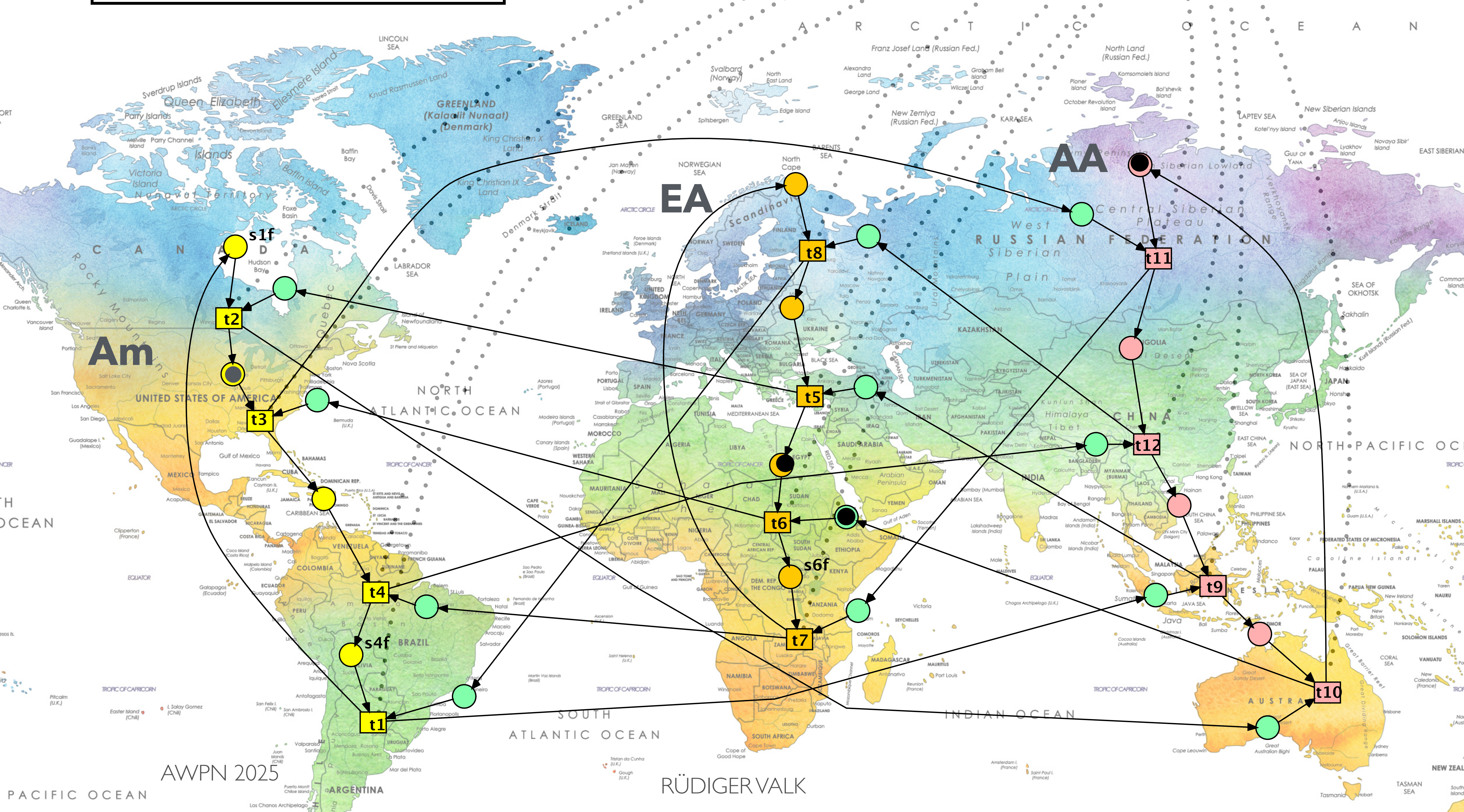

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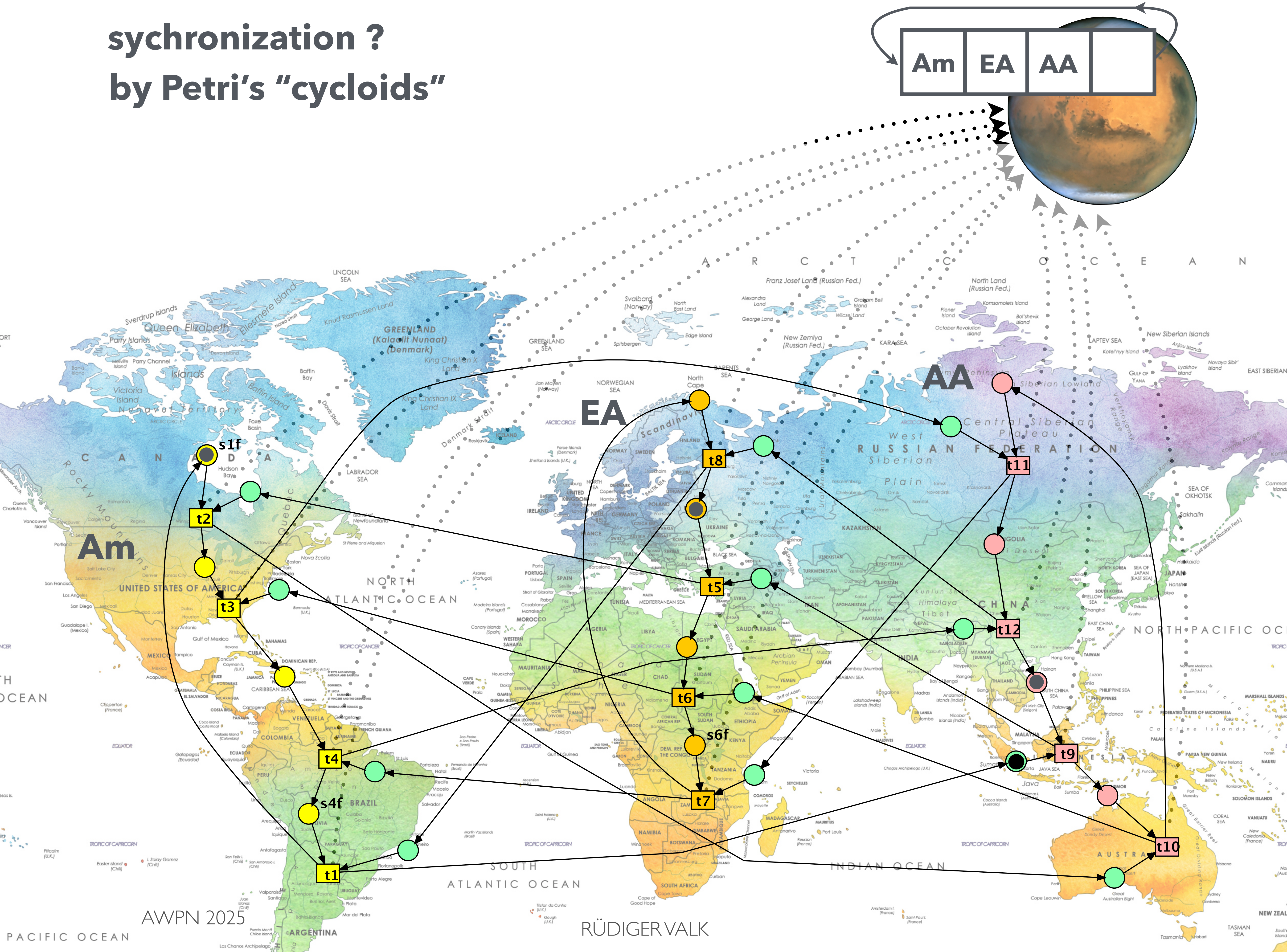
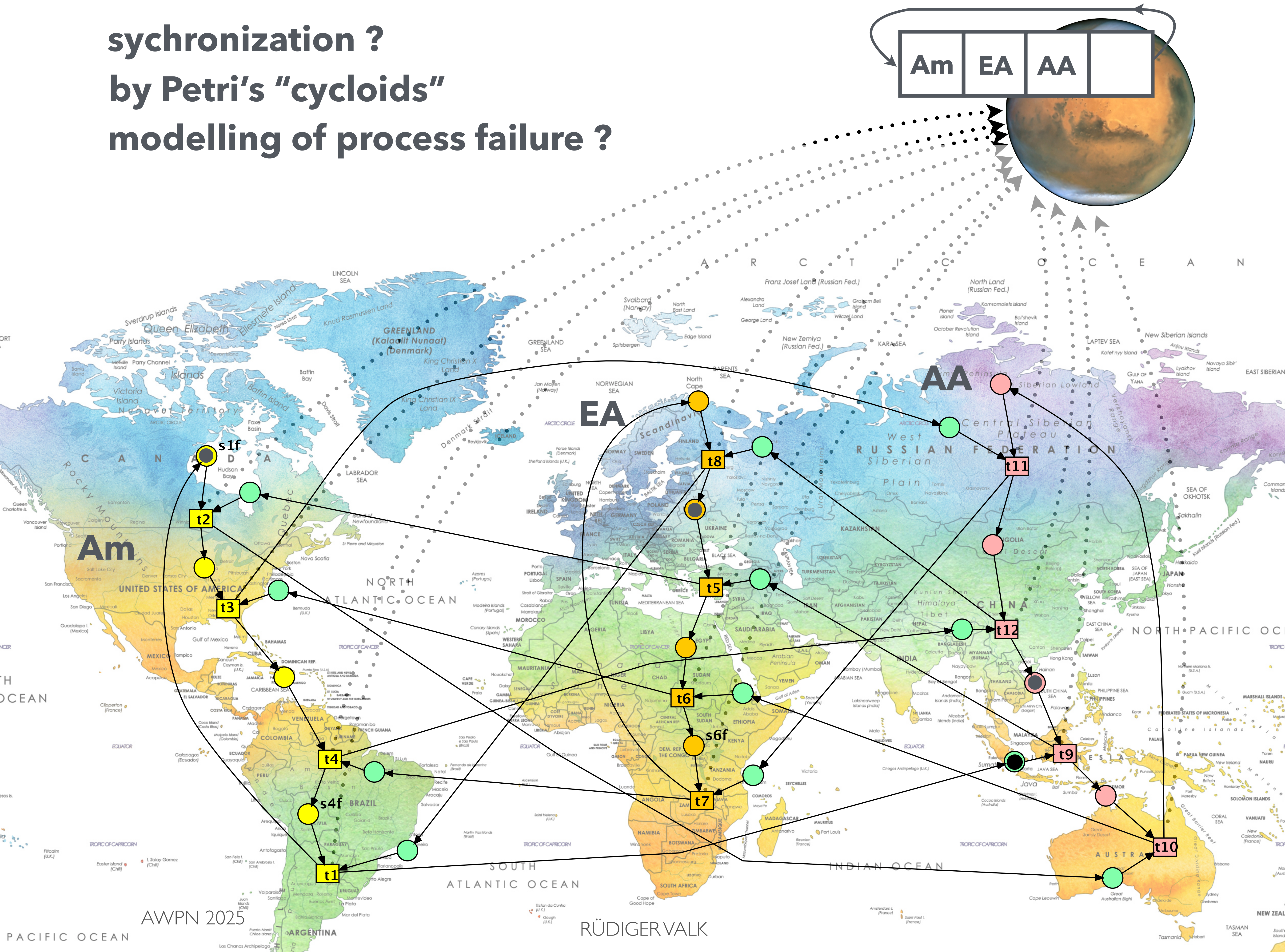
by Petri's "cycloids"



sychronization ?
by Petri's "cycloids"
modelling of process failure ?



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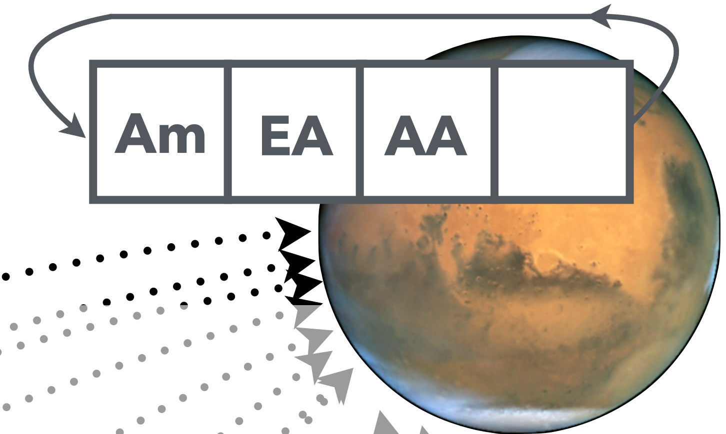
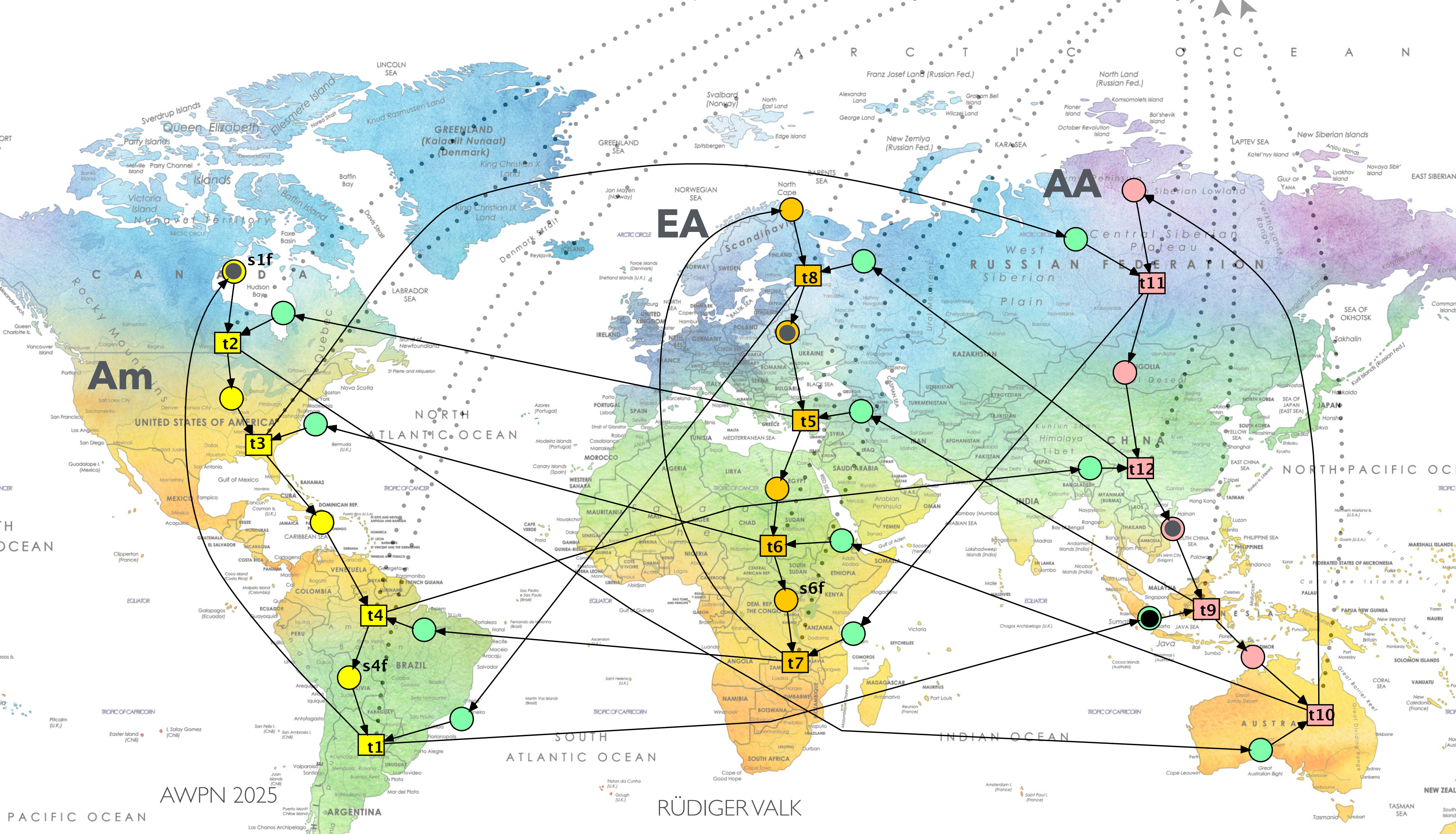
sychronization ?

by Petri's "cycloids"

modelling of process failure ?

by removing the control token





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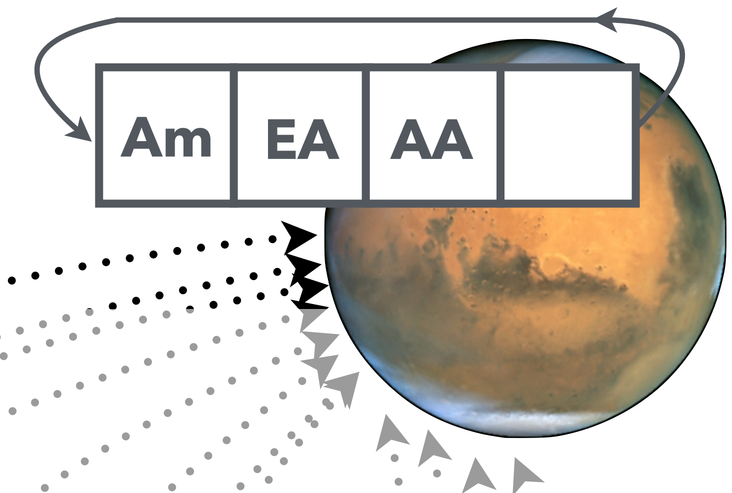

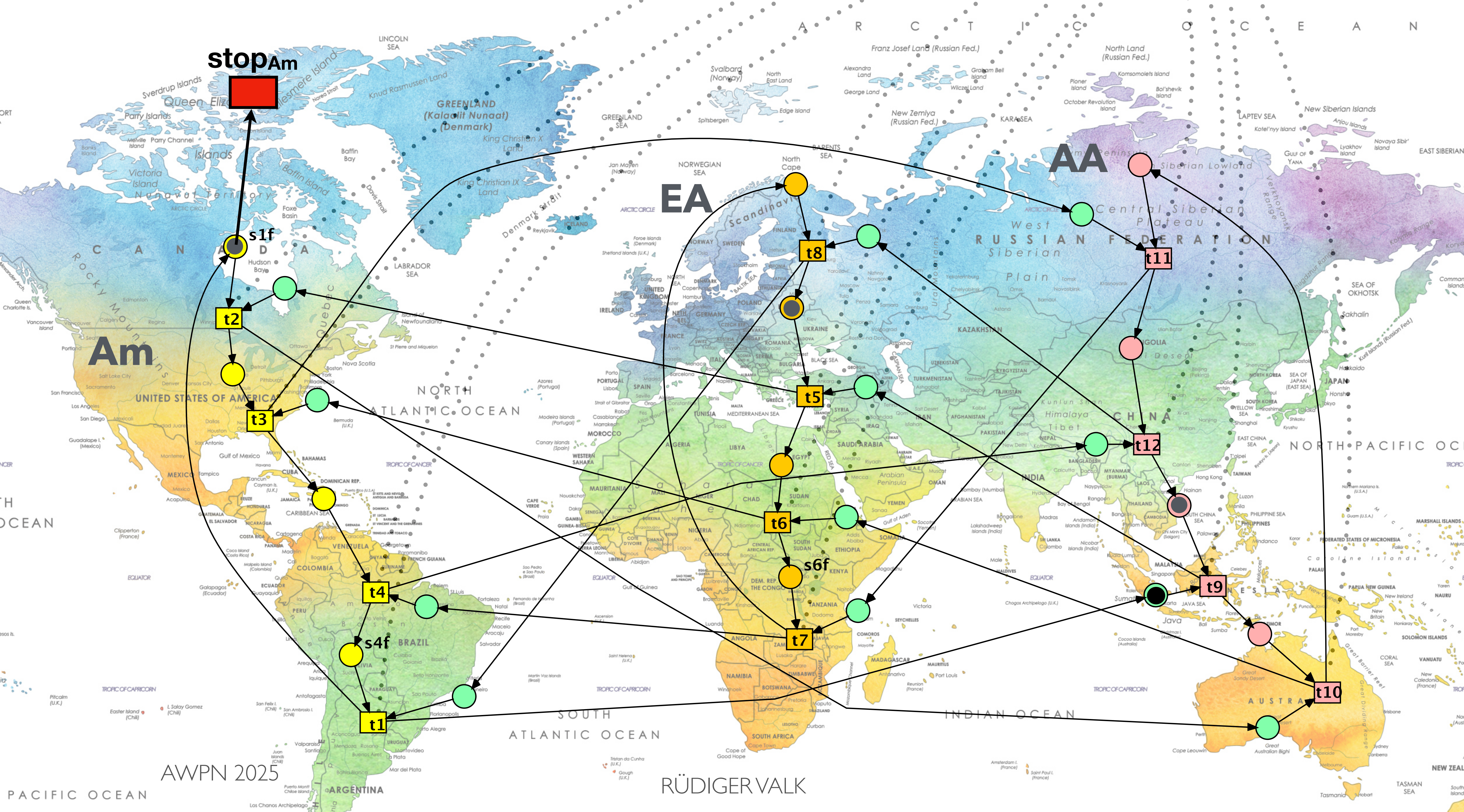

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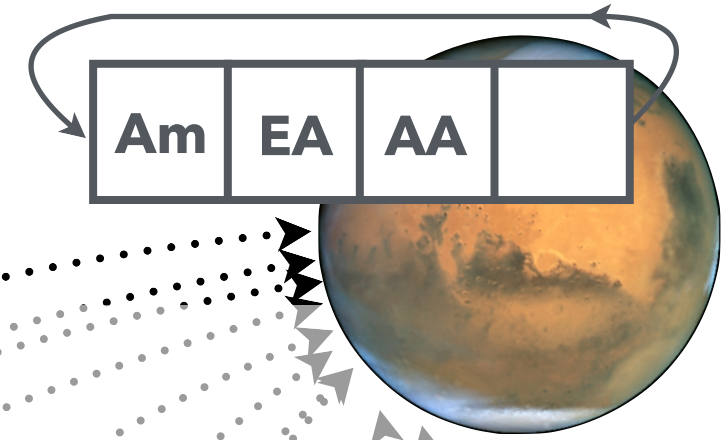
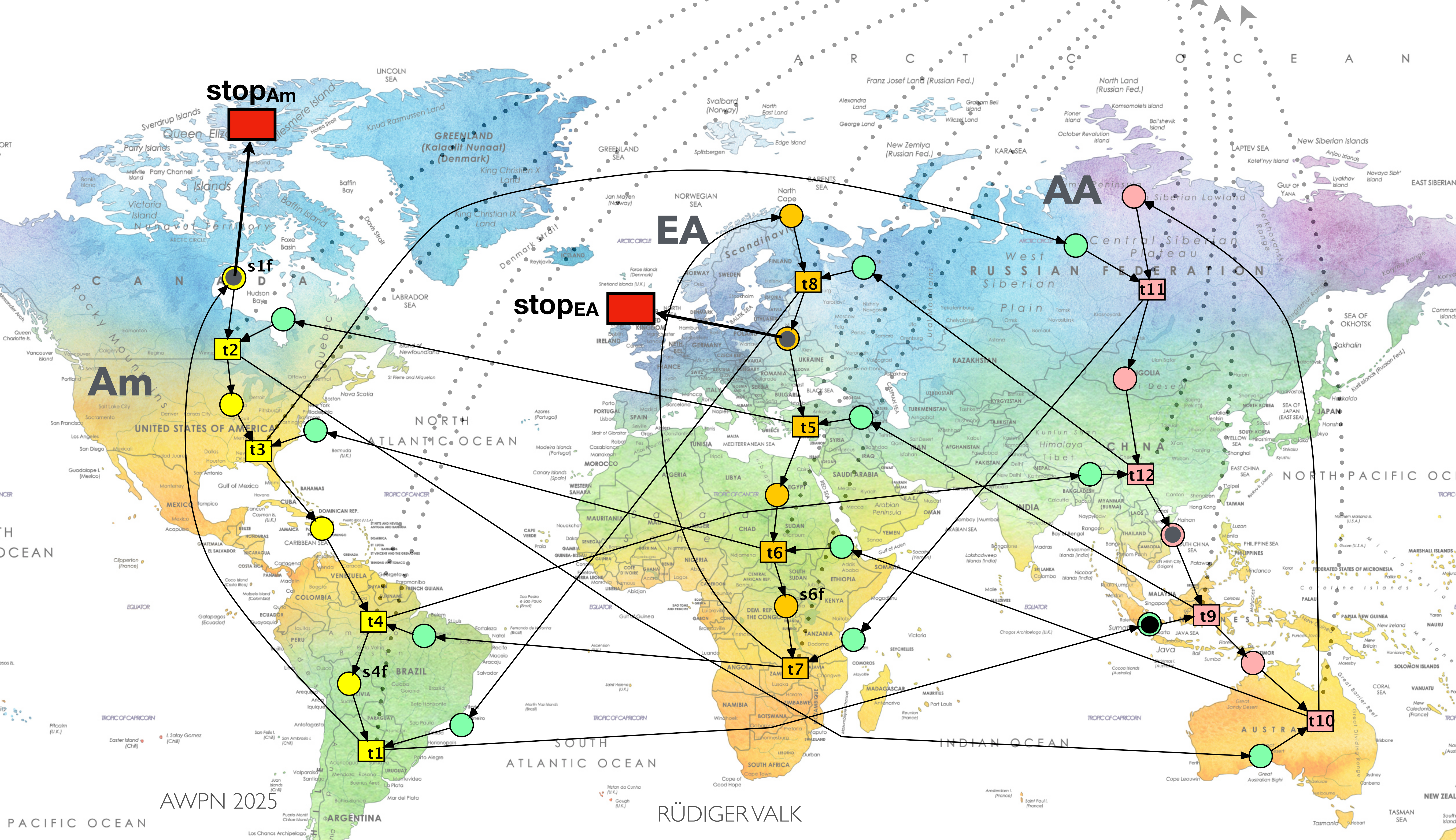
sychronization ?

by Petri's "cycloids"

modelling of process failure ?

by removing the control token



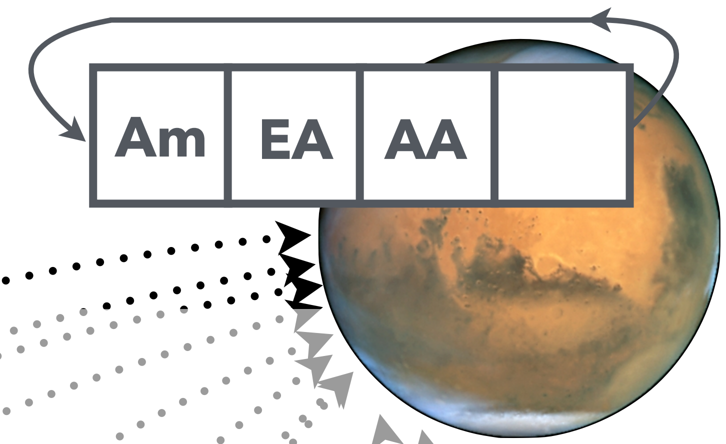


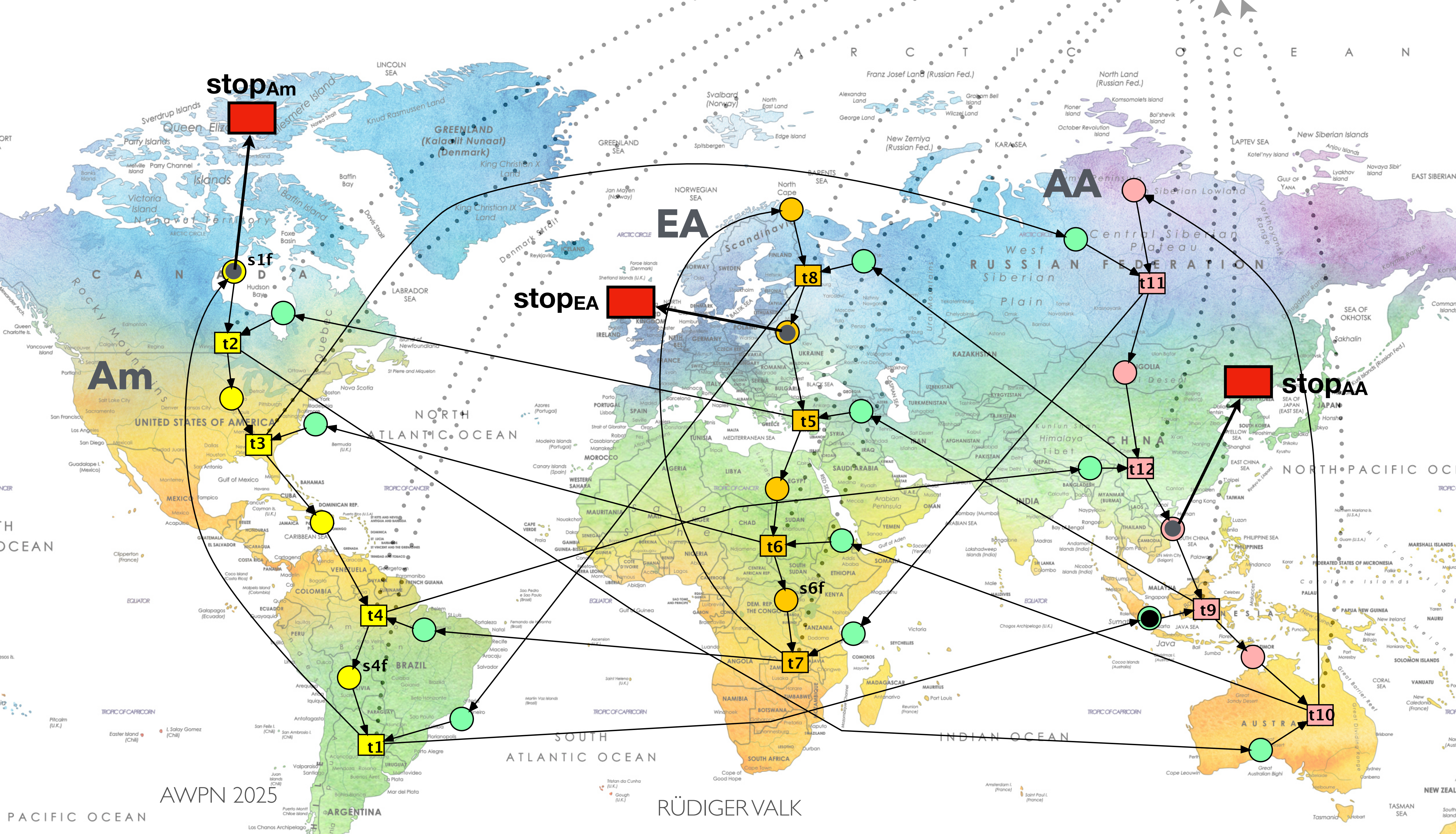
sychronization ?

by Petri's "cycloids"

modelling of process failure ?

by removing the control token



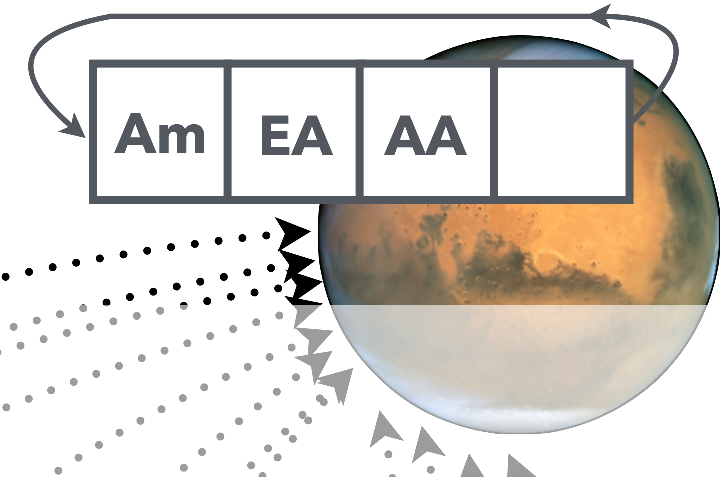


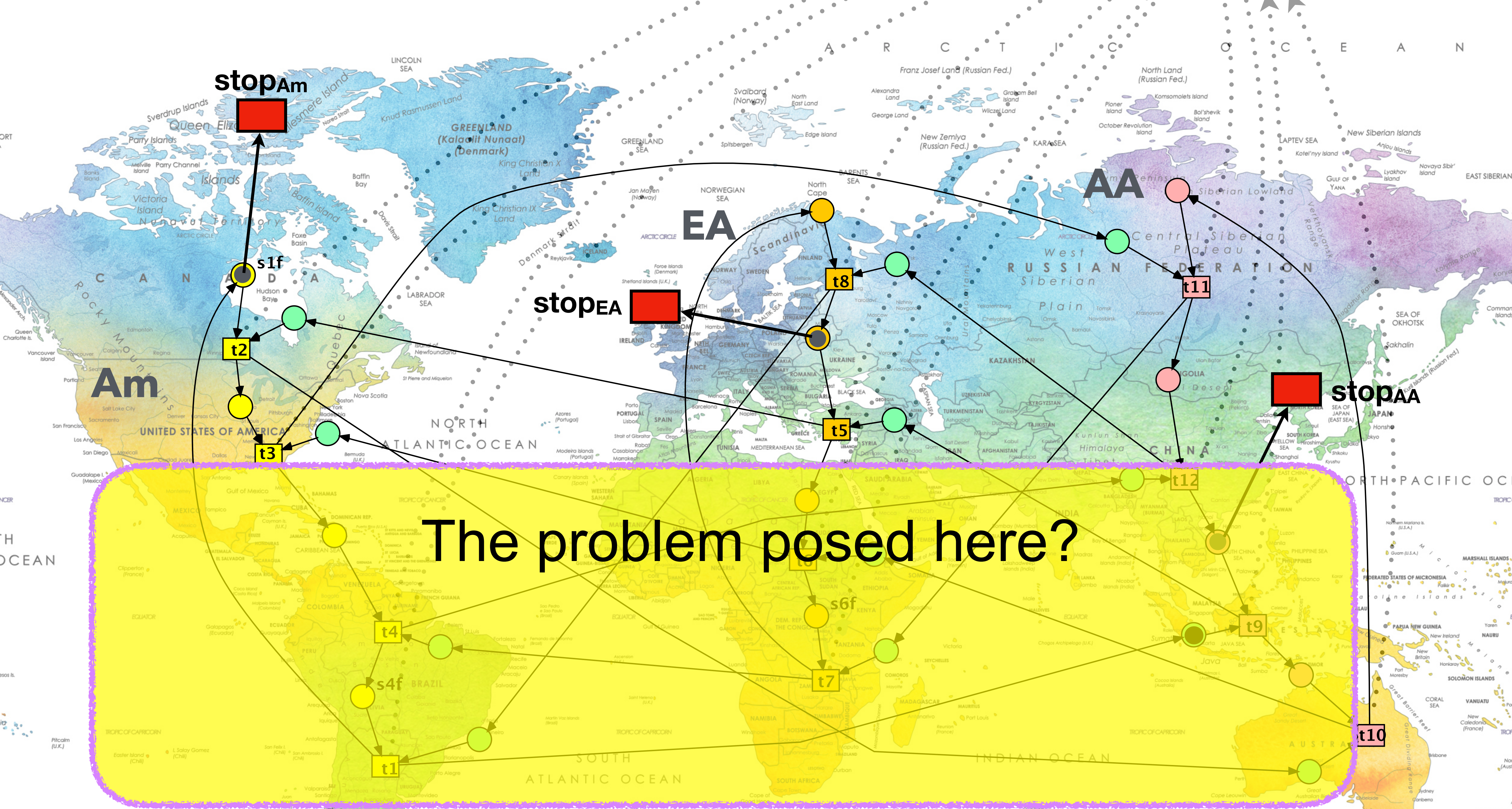
sychronization ?

by Petri's "cycloids"

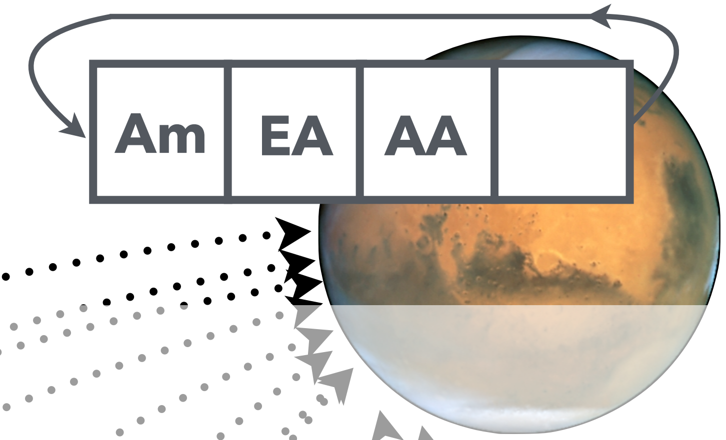
modelling of process failure ?

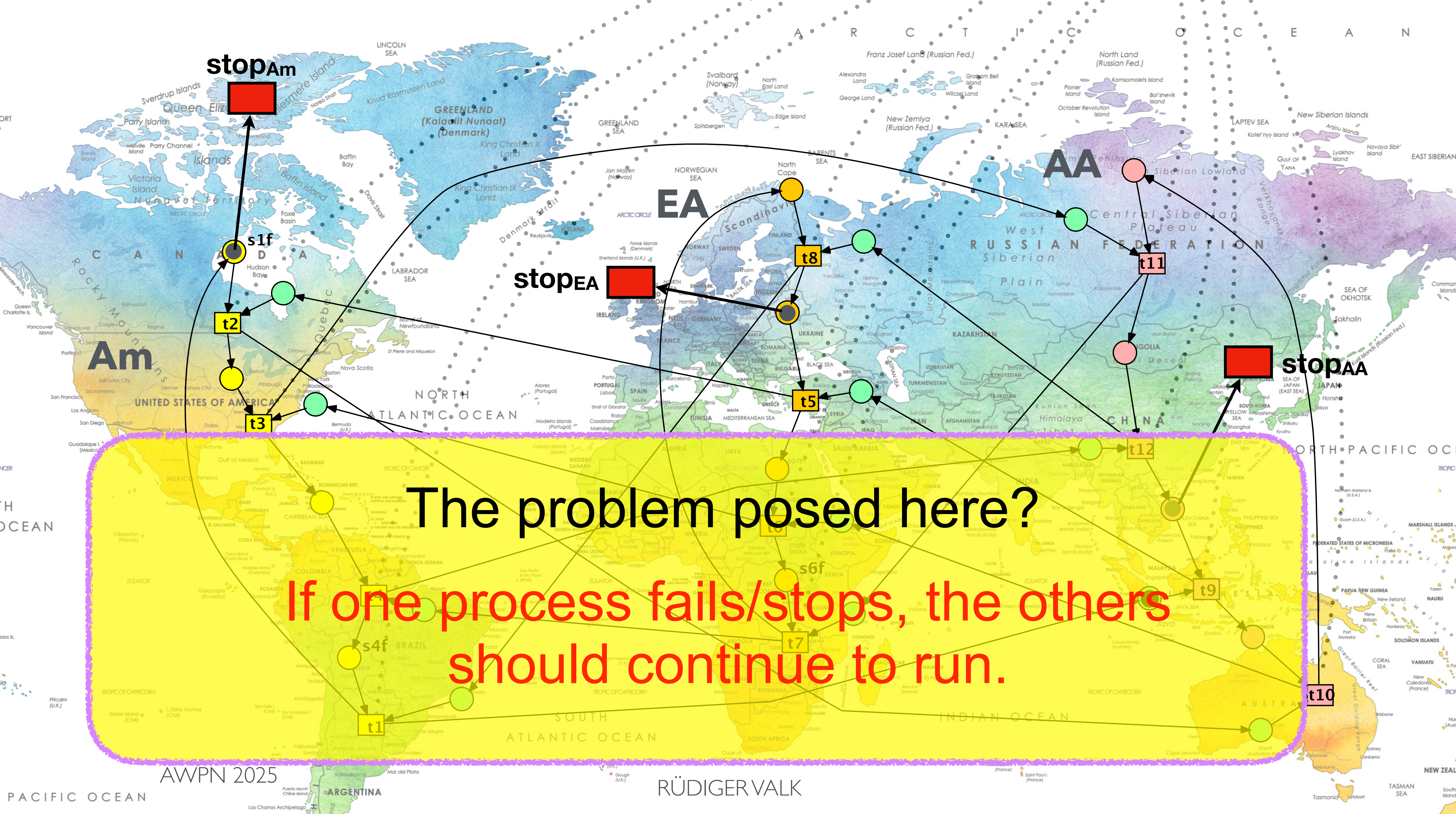
by removing the control token





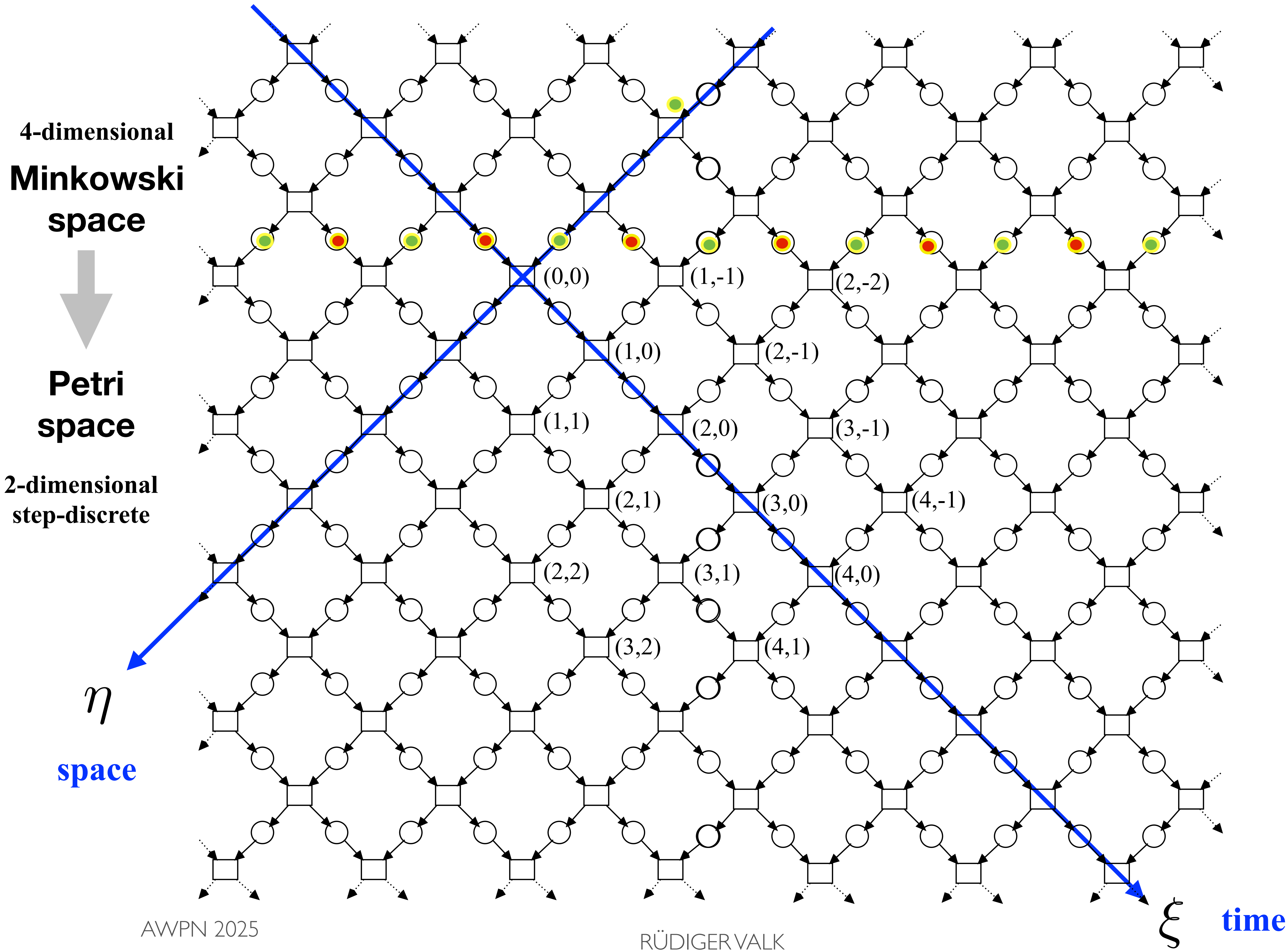
sychronization ?
by Petri's "cycloids"
modelling of process failure ?
by removing the control token





The problem posed here?

If one process fails/stops, the others should continue to run.



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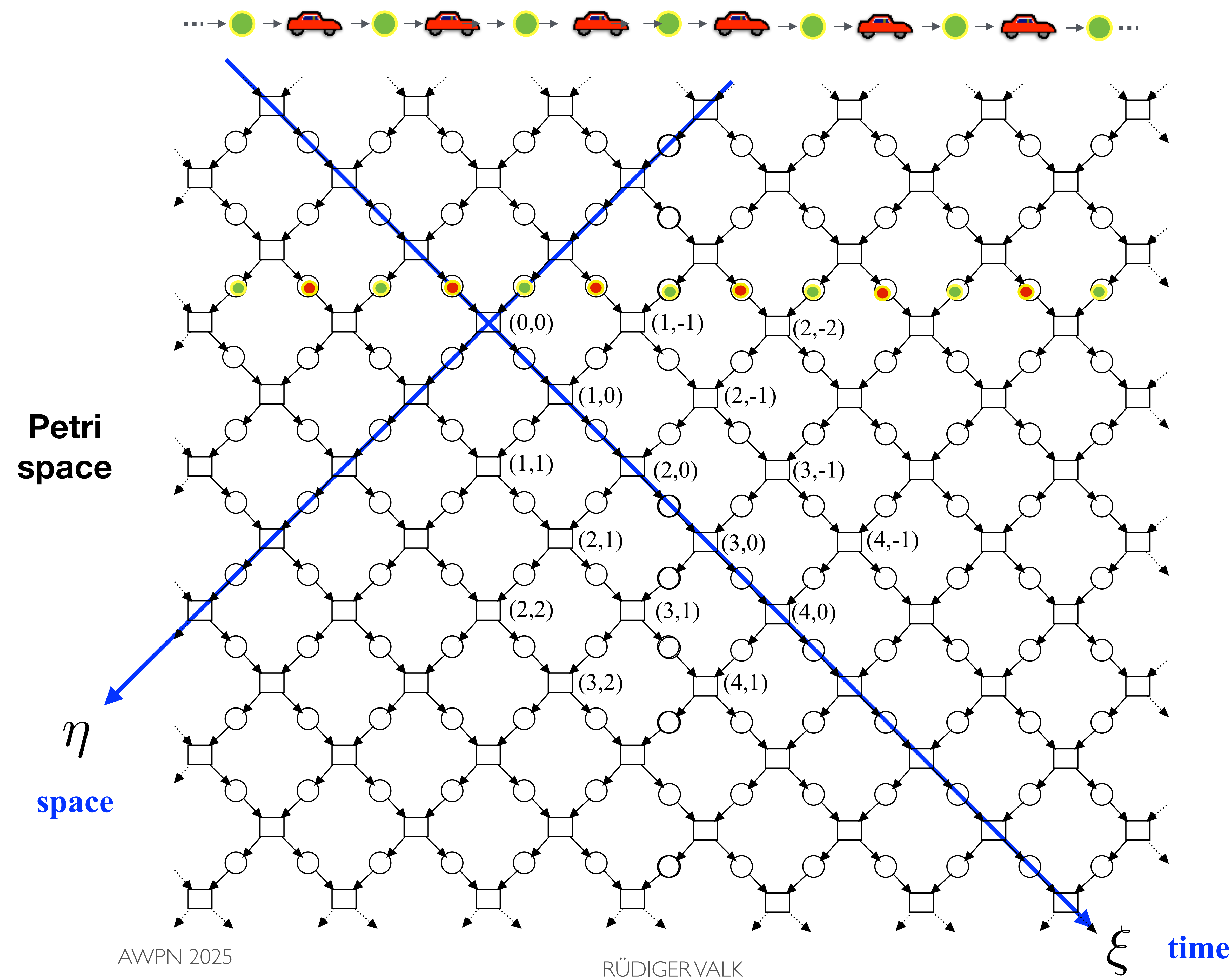
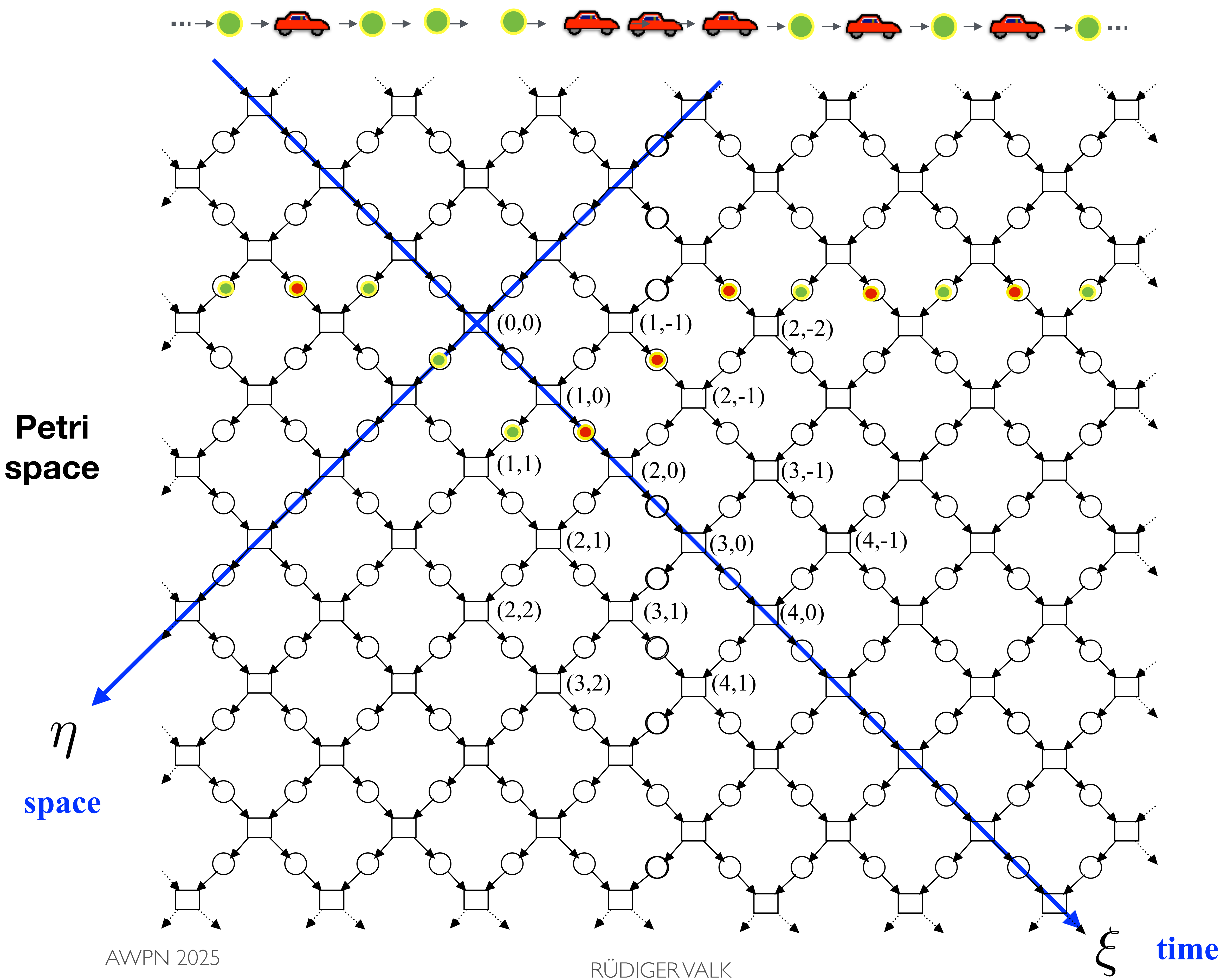


Petri
space
(0,0)
(1,-1)
(2,-2)
(1,0)
(2,-1)
(1,1)
(2,0)
(3,-1)
(2,1)
(3,0)
(4,-1)
(2,2)
(3,1)
(4,0)
(3,2)
(4,1)
η
space
ξ
time


Petri
space
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(2,-1)
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(2,1)
(3,0)
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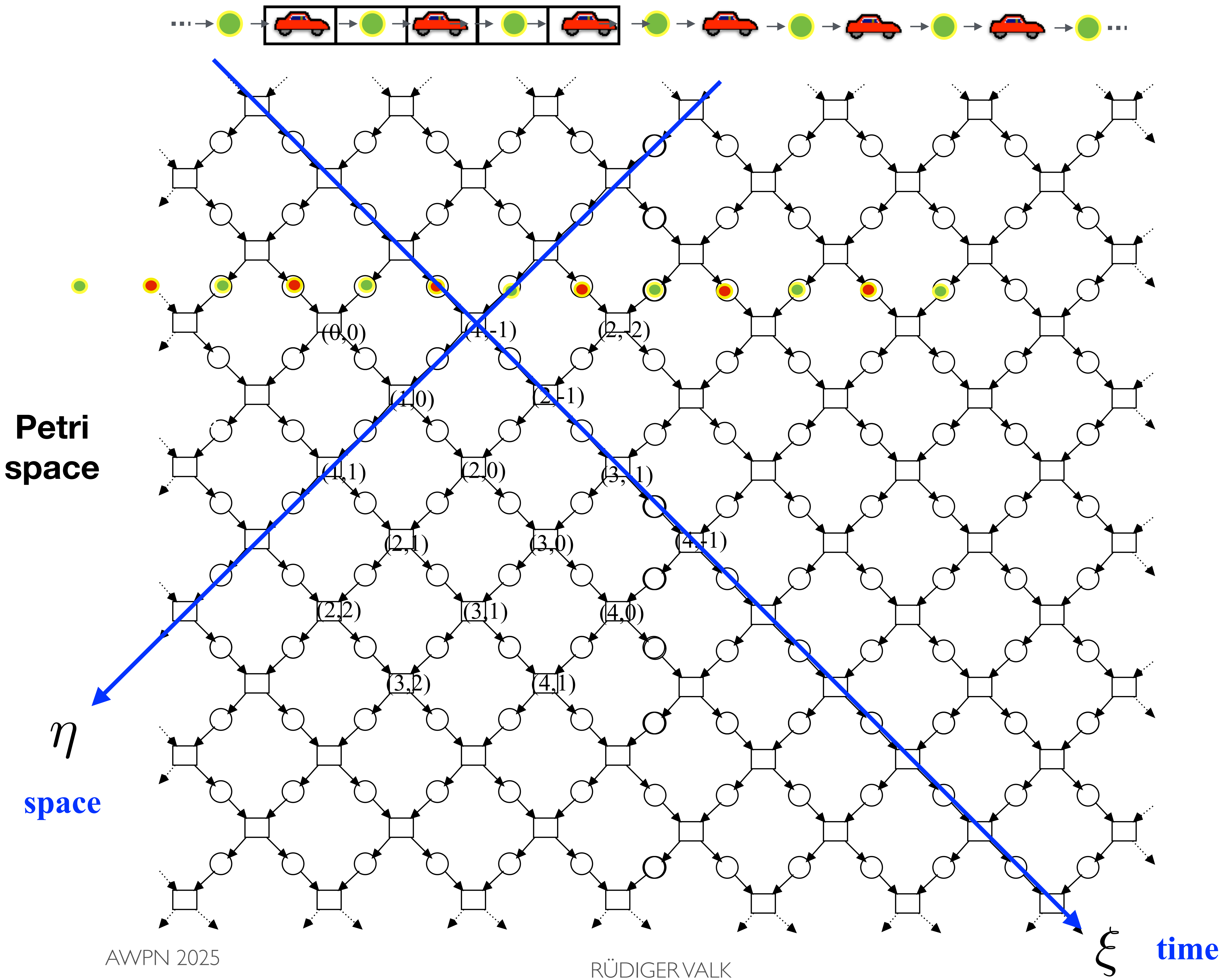
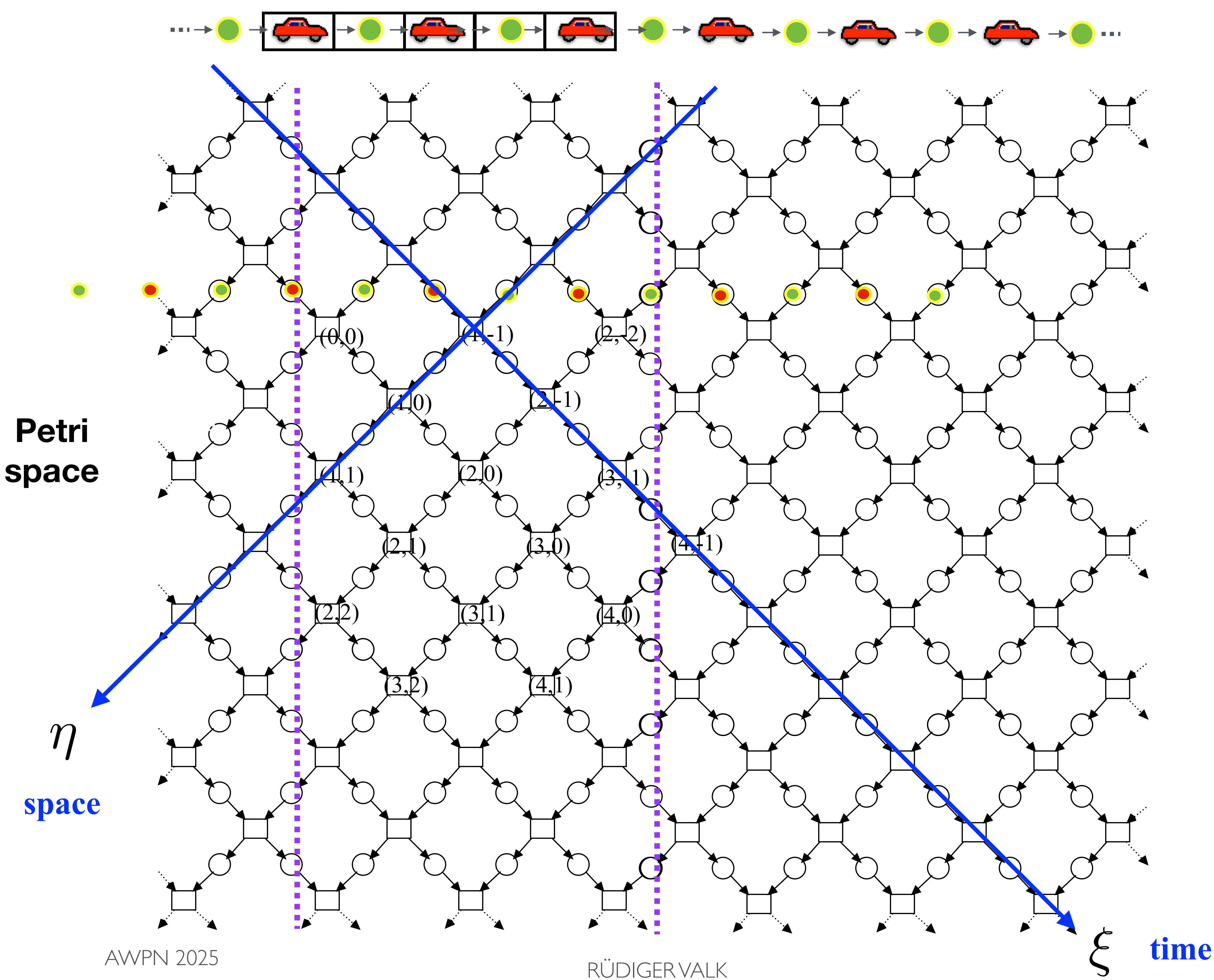
(2,2)
(3,1)
(4,0)
(3,2)
(4,1)
η
space
ξ
time


Petri
space
(0,0)
(1,-1)
(2,-2)
(1,0)
(2,-1)
(1,1)
(2,0)
(3,-1)
(2,1)
(3,0)
(4,-1)
(2,2)
(3,1)
(4,0)
(3,2)
(4,1)
η
space
ξ
time



Petri
space
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(1,1)
(2,0)
(3,-1)
(2,1)
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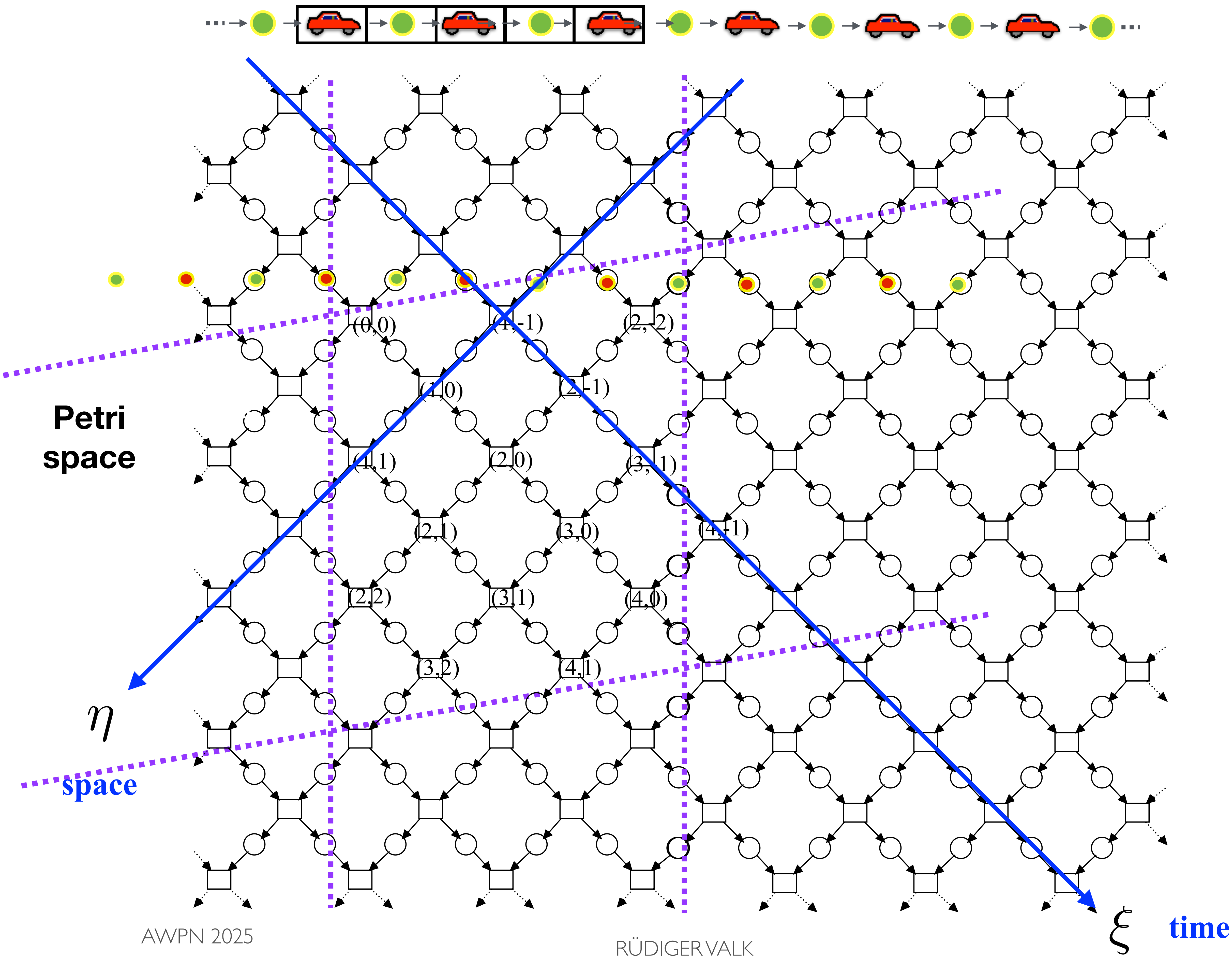
(2,2)
(3,1)
(4,0)
(3,2)
(4,1)
η
space
ξ
time



Petri
space
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(1,1)
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(2,1)
(3,0)
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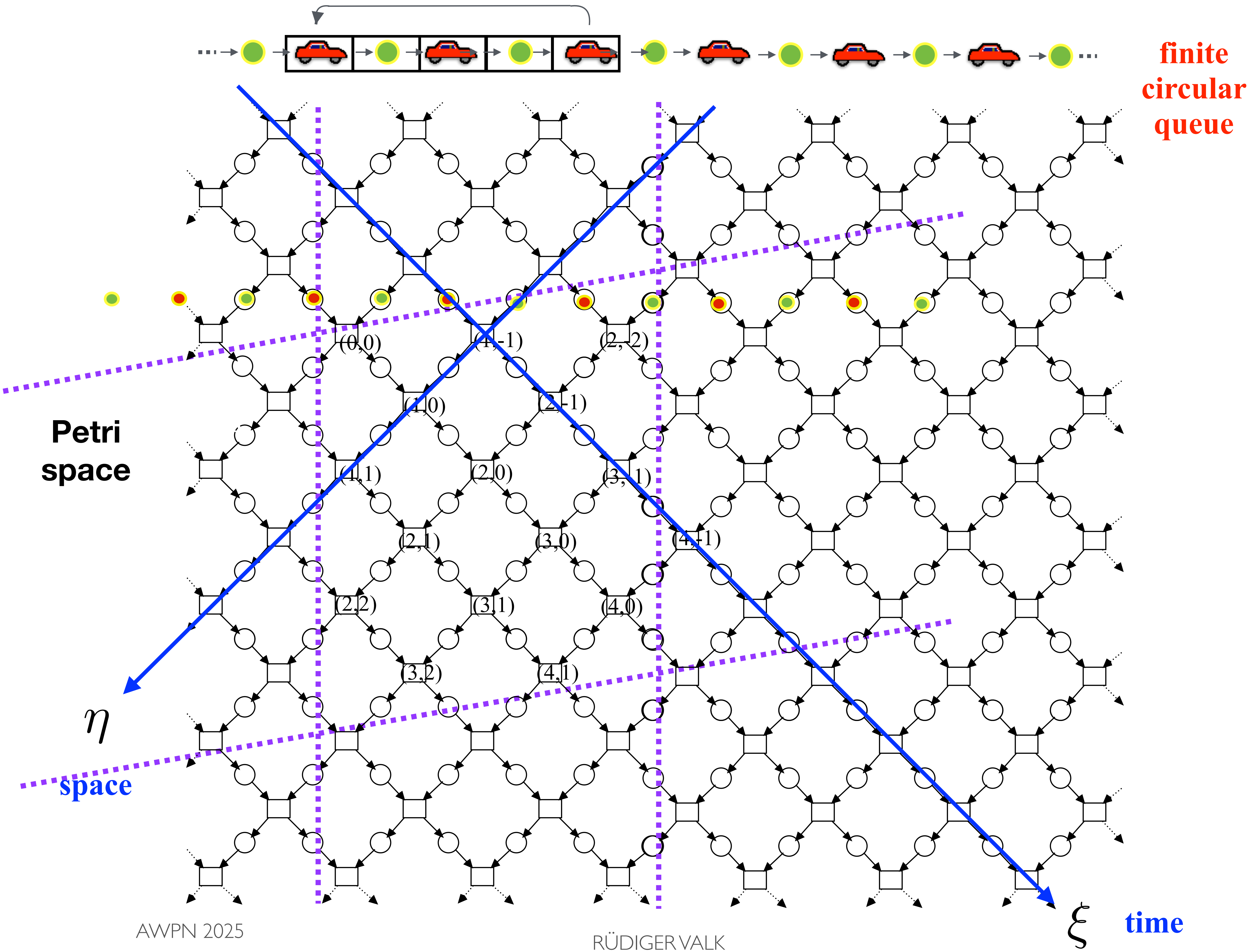
(2,2)
(3,1)
(4,0)
(3,2)
(4,1)
η
space
ξ
time


finite
circular
queue
Petri
space
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(2,1)
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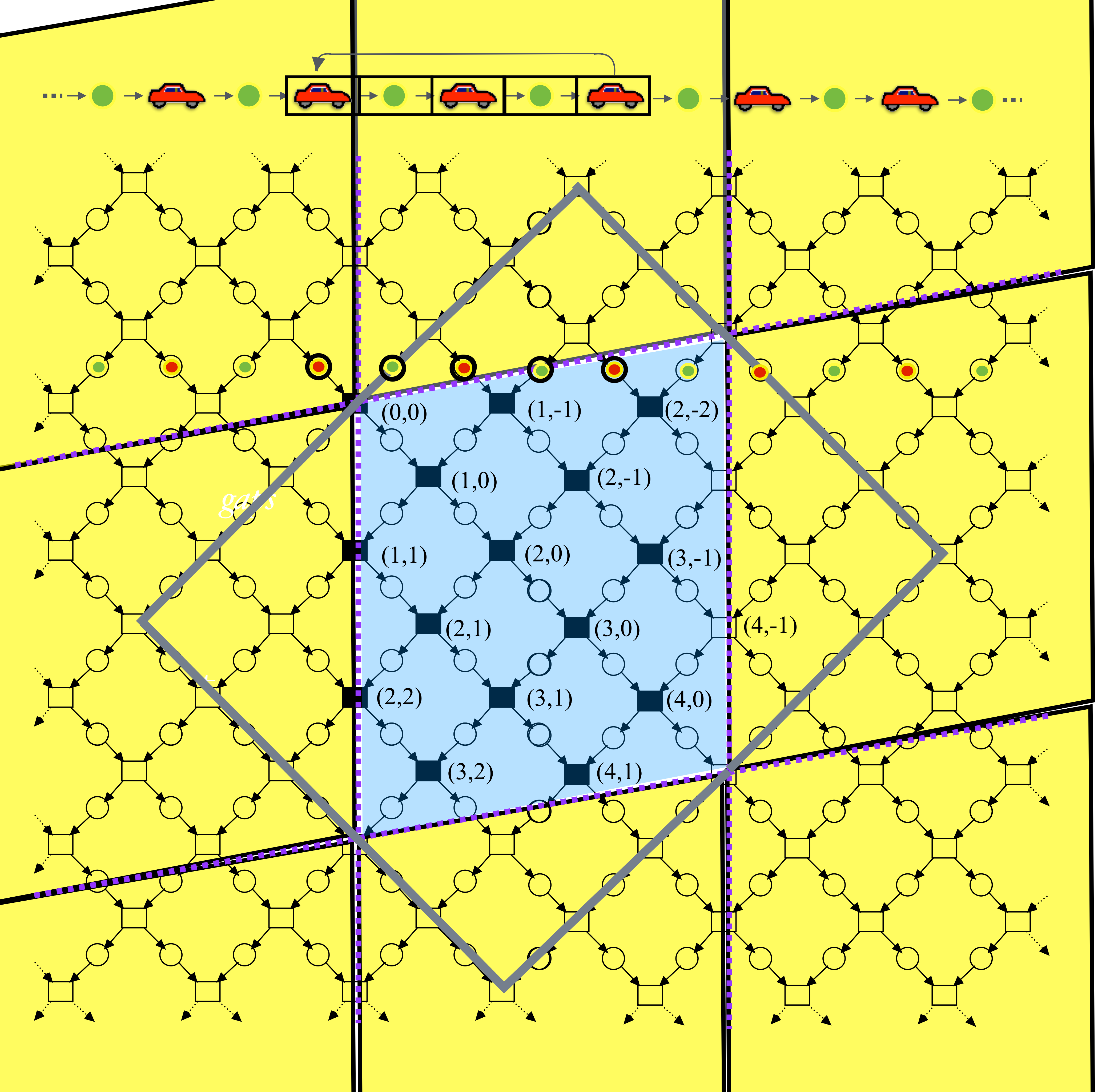
(3,1)
(4,0)
(3,2)
(4,1)
η
space
ξ
time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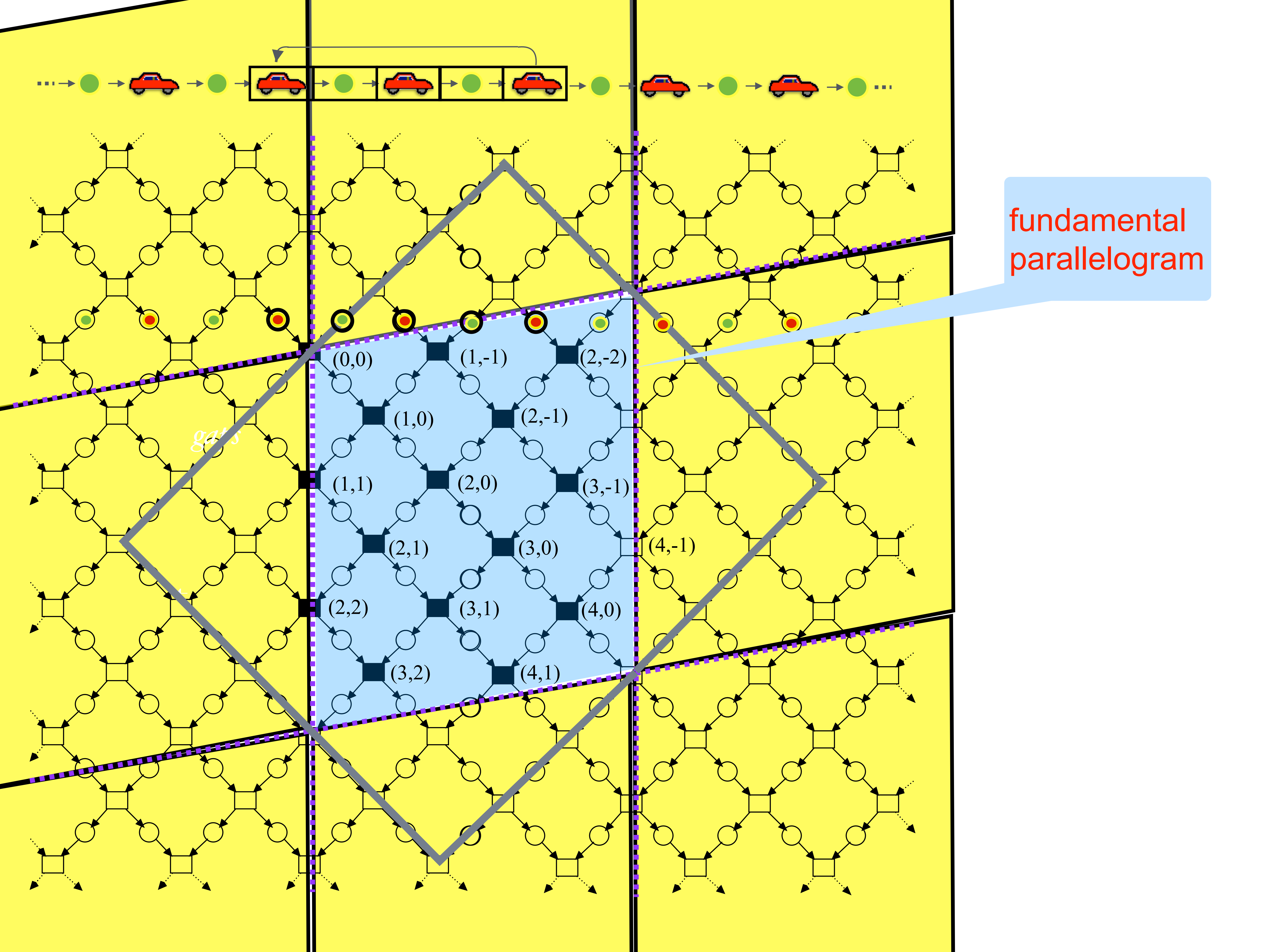
(4,0)
(3,2)
(4,1)



fundamental parallelogram
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(2,0)
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(2,1)
(3,0)
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(2,2)
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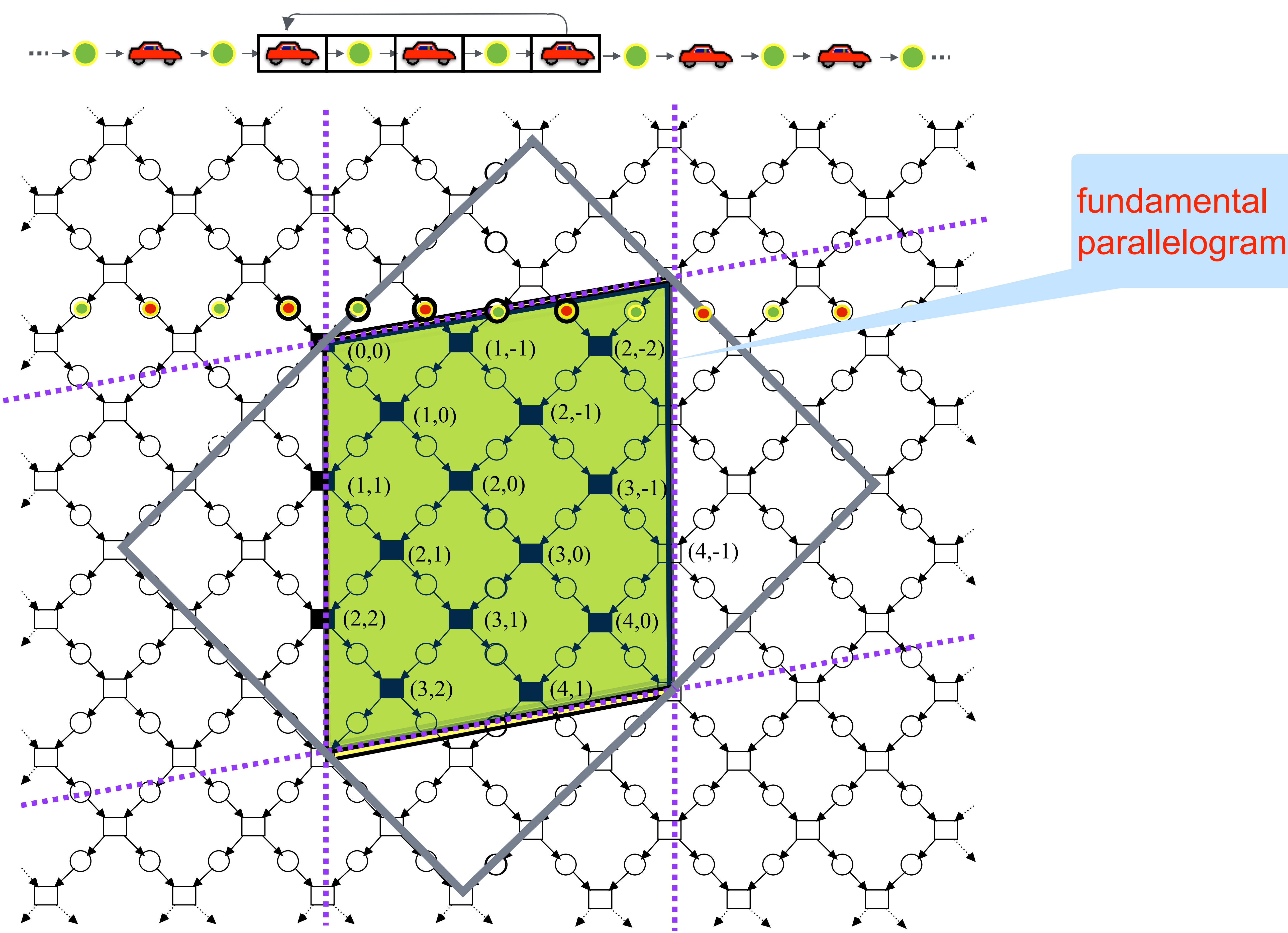
(4,0)
(3,2)
(4,1)



fundamental
parallelogram
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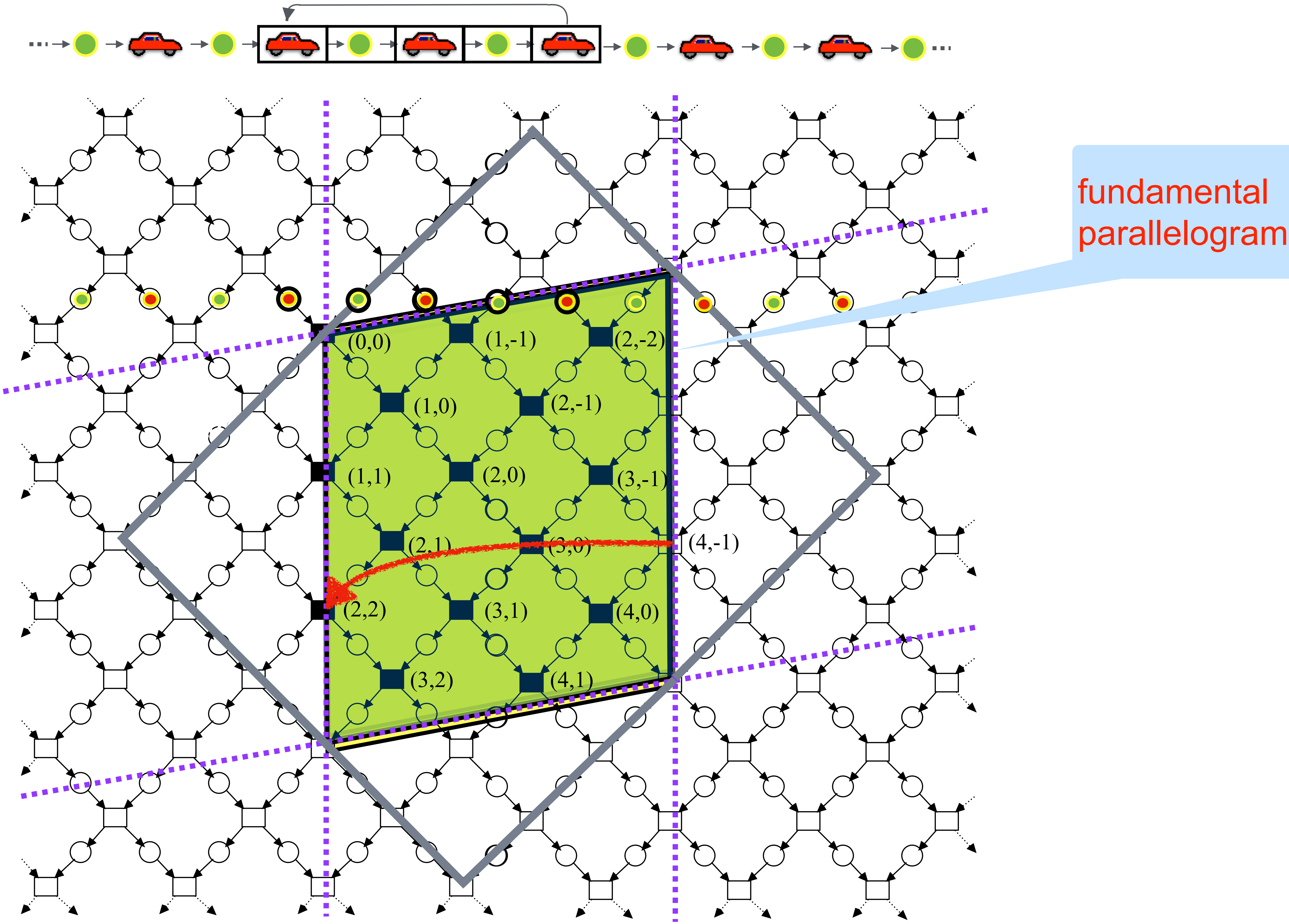
(3,2)
(4,1)



fundamental
parallelogram
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(3,0)
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(3,1)
(4,0)
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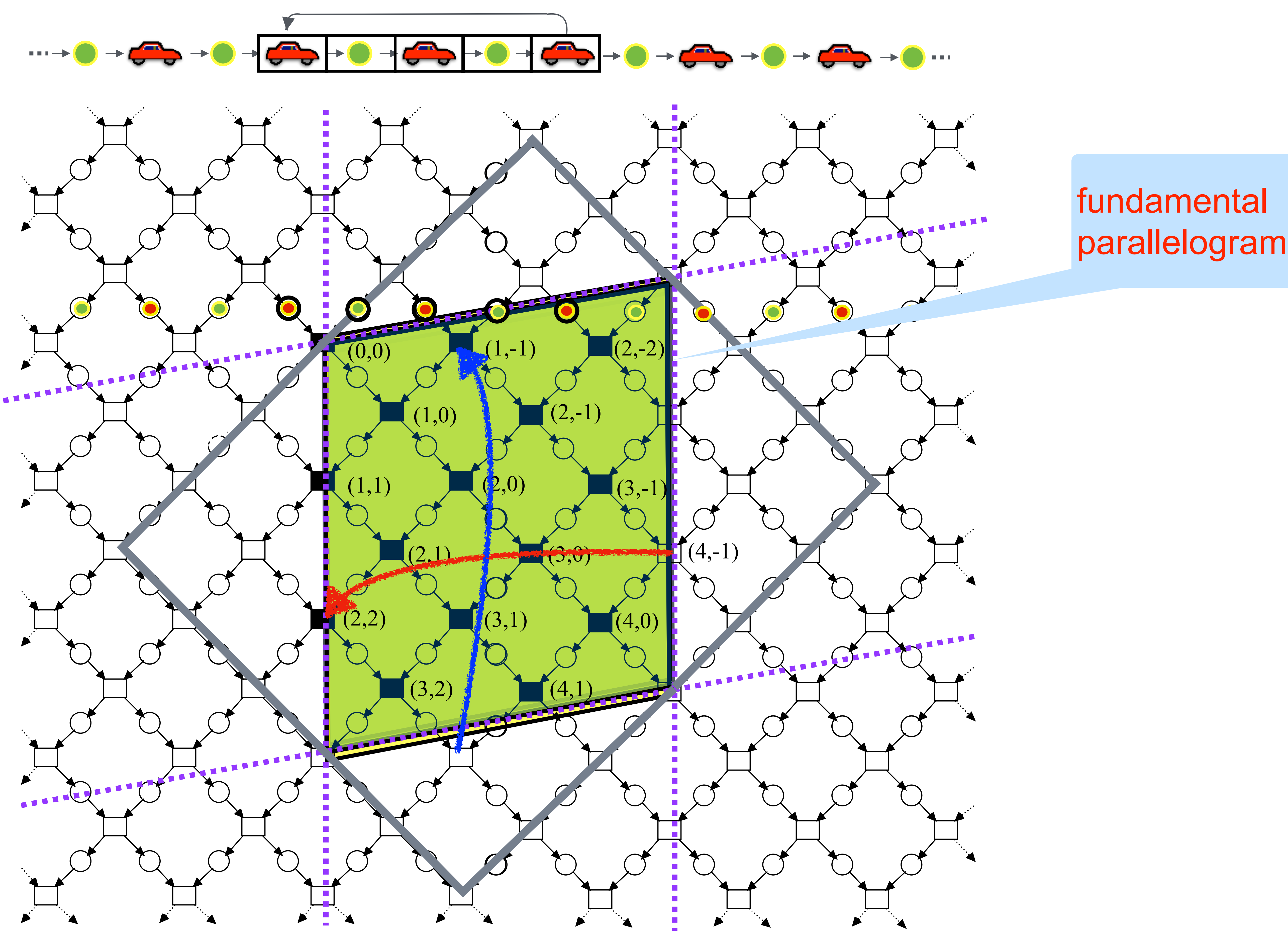
(4,1)


fundamental
parallelogram
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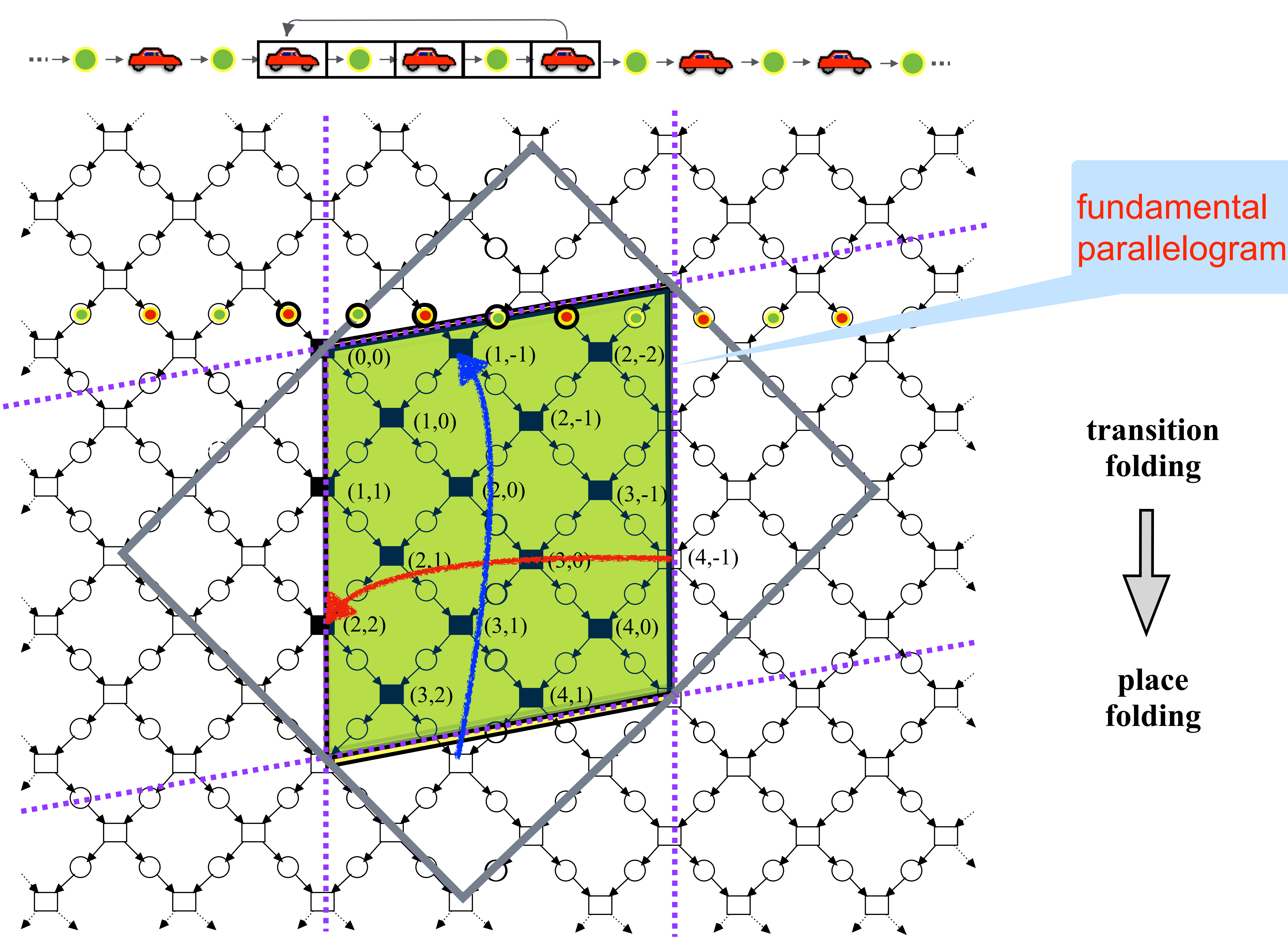
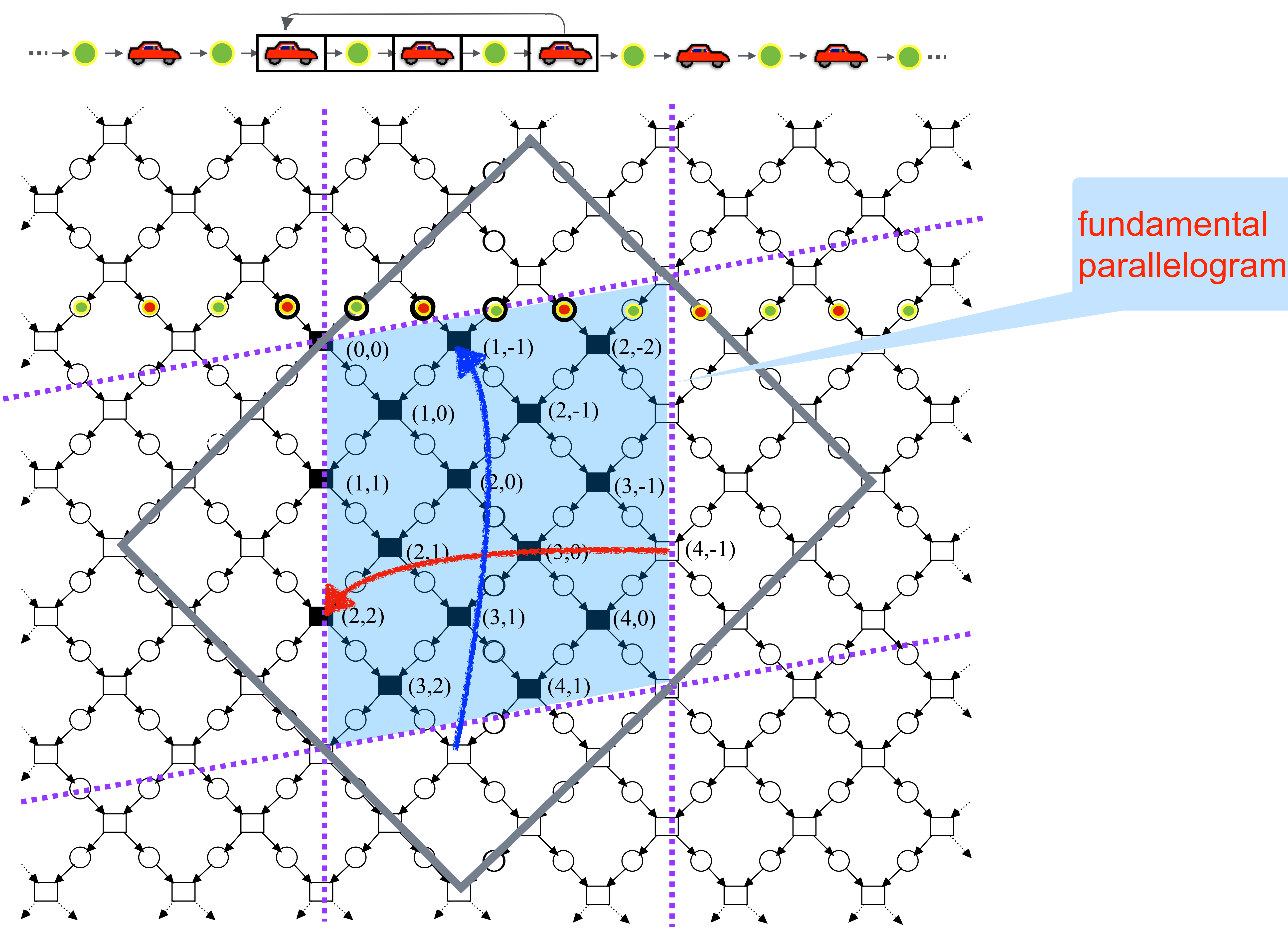


Place Foldings of Regular Cycloids for Modelling StopTolerance



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(2,2)
(3,1)
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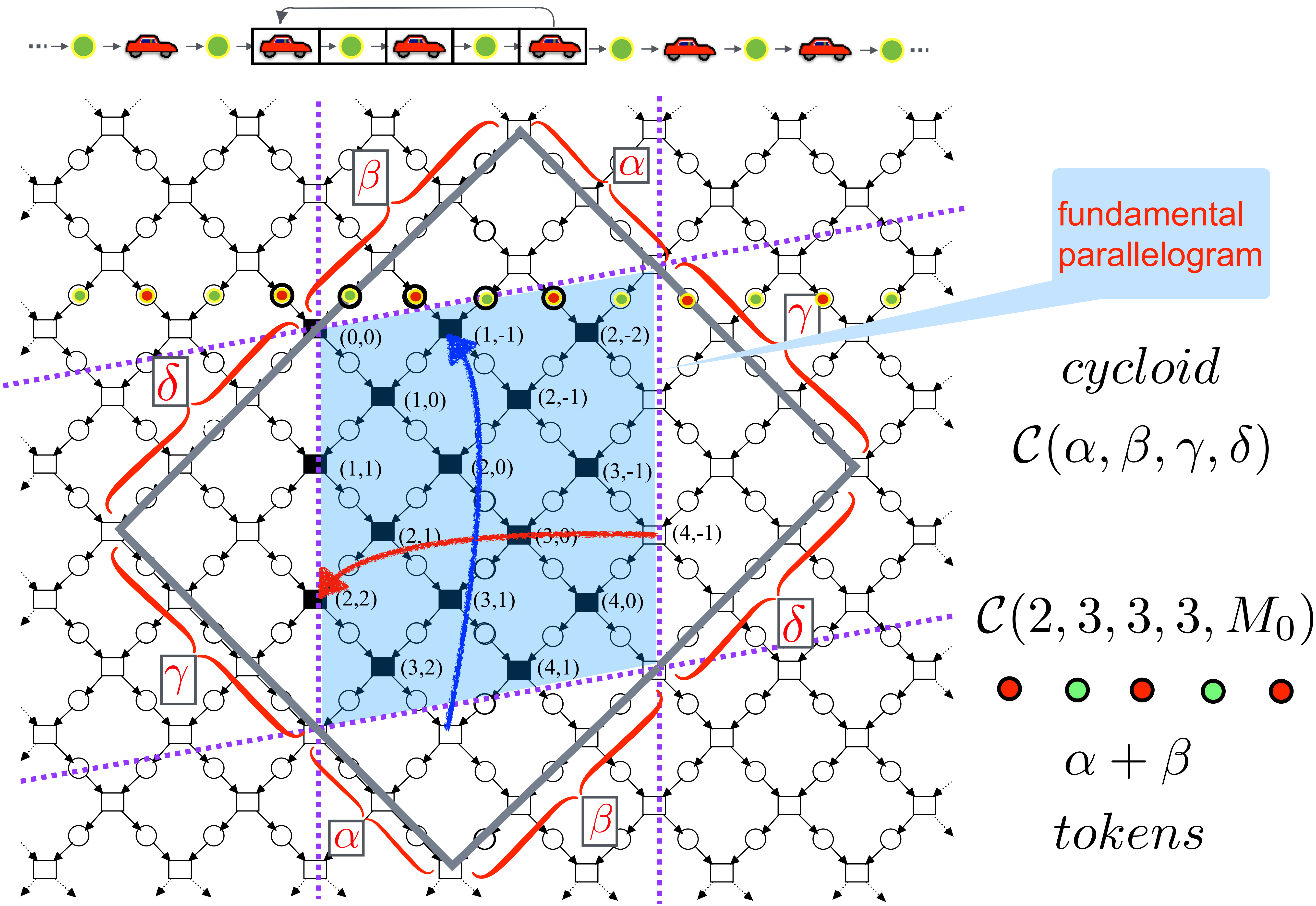
(3,2)
(4,1)
fundamental parallelogram



fundamental parallelogram
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(1,-1)
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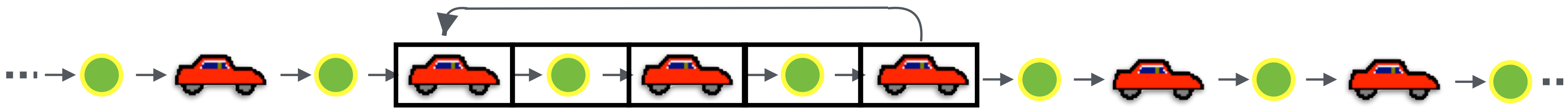
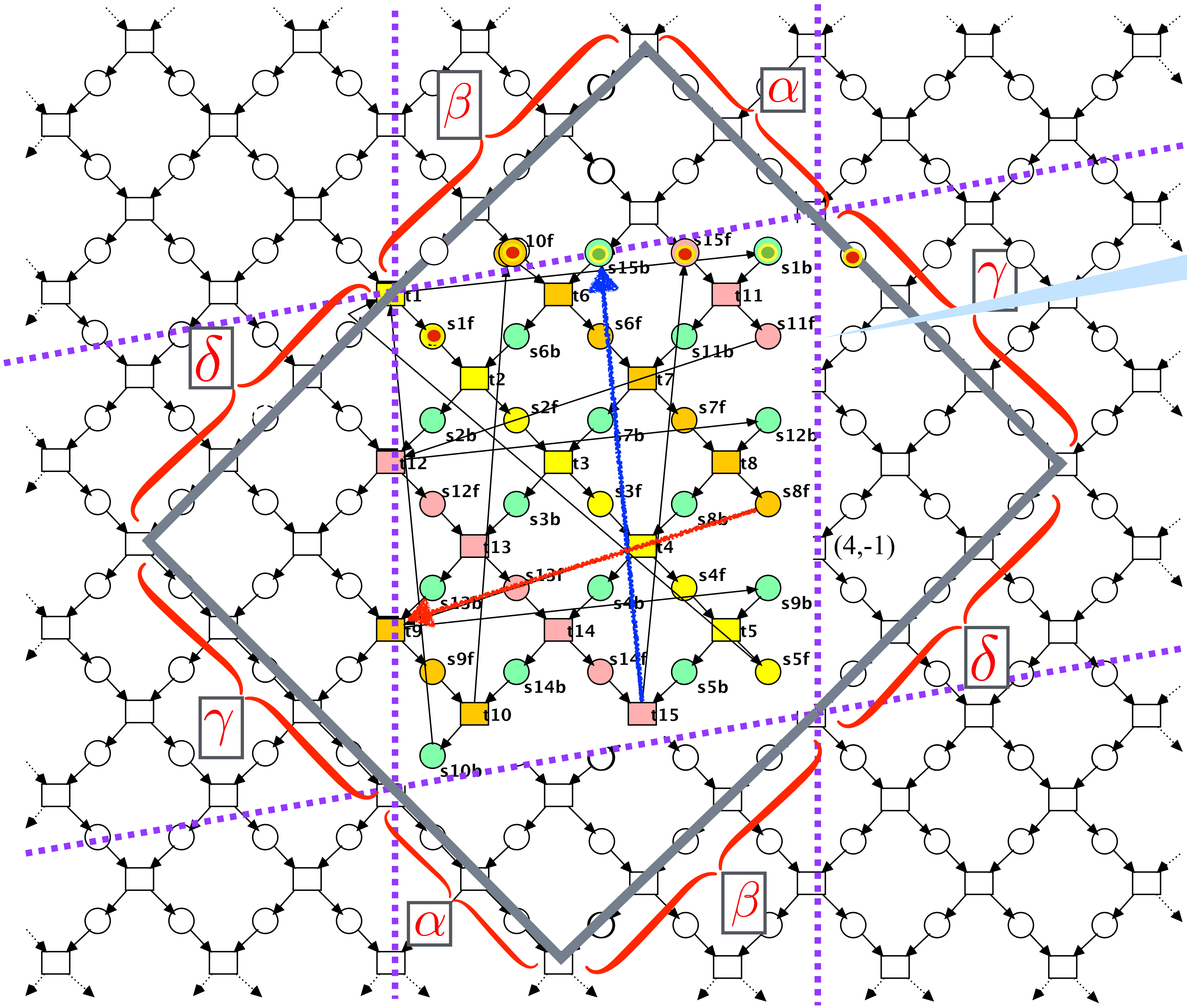
(2,-1)
(1,1)
(2,0)
(3,-1)
(2,1)
(3,0)
(4,-1)
(2,2)
(3,1)
(4,0)
(3,2)
(4,1)
α
β
γ
δ
cycloid
$\mathcal{C}(\alpha, \beta, \gamma, \delta)$
$\mathcal{C}(2,3,3,3,M_0)$
$\alpha + \beta$
tokens





fundamental parallelogram

cycloid

$$\mathcal{C}(\alpha, \beta, \gamma, \delta)$$

$$\mathcal{C}(2, 3, 3, 3, M_0)$$

$$\alpha + \beta$$

tokens

Cycloids are characterised by 4 parameters of natural numbers.

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) \qquad \alpha, \beta, \gamma, \delta \in \mathbb{N}_+$$

the canonical cycloid

$$\beta = \gamma = \delta$$

Cycloids are characterised by 4 parameters of natural numbers.

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) \qquad \alpha, \beta, \gamma, \delta \in \mathbb{N}_+$$

$$= \mathcal{C}(2, 3, 3, 3)$$

number of gaps

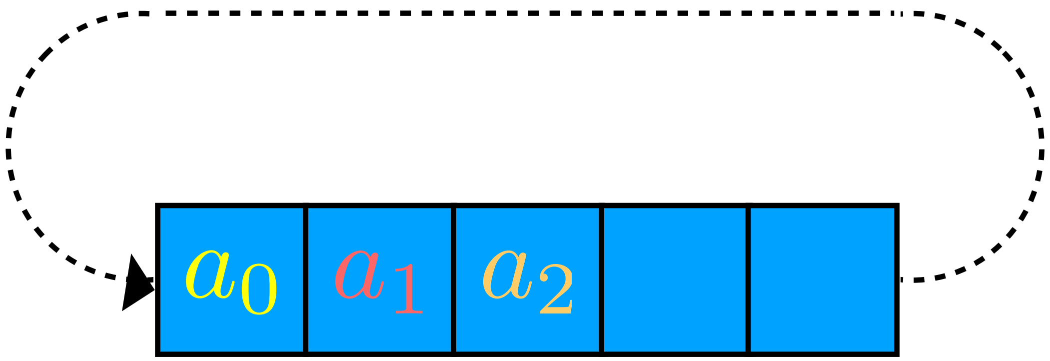
number of processes

the canonical cycloid

$$\beta = \gamma = \delta$$

circular traffic queues of process length 5 and 8

circular queue of 3 items and 2 gaps



$$\mathcal{C}(\alpha, \beta, \gamma, \delta)$$
$$= \mathcal{C}(2, 3, 3, 3)$$

number of gaps

number of processes

the canonical cycloid

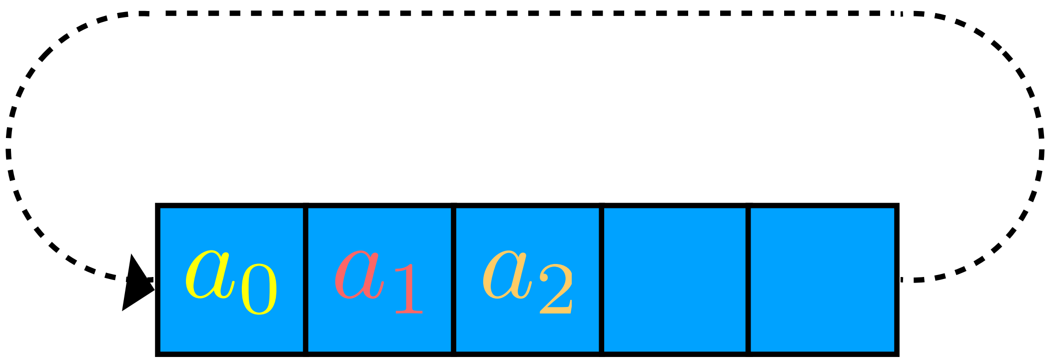
$$\beta = \gamma = \delta$$

the regular cycloid

β devides δ

circular traffic queues of process length 5 and 8

circular queue of 3 items and 2 gaps



$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 3, 3, 3)$$

number of gaps

number of processes

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 3, 4, 6)$$

number of processes

the canonical cycloid

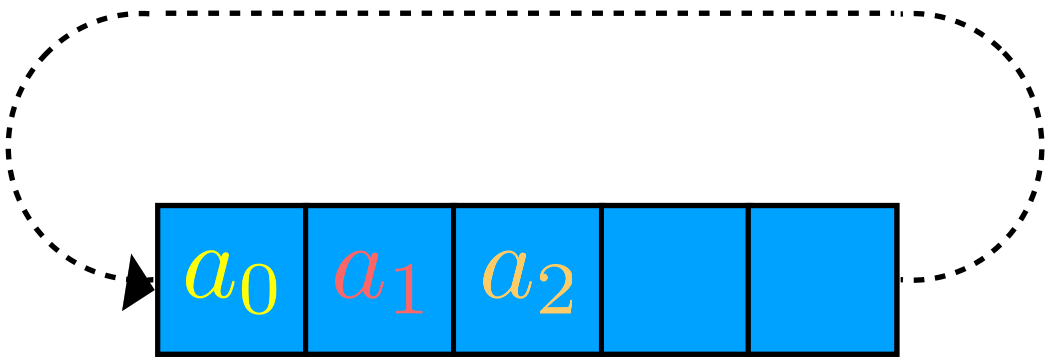
$$\beta = \gamma = \delta$$

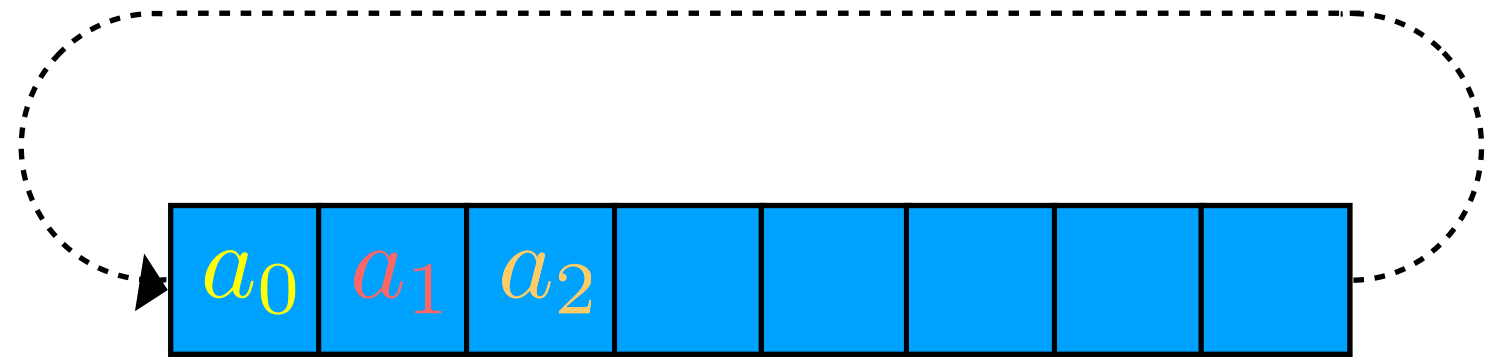
the regular cycloid

β devides δ

circular traffic queues of process length 5 and 8

circular queue of 3 items and 2 gaps





length of processes: $p = \frac{A}{\beta} = \frac{24}{3} = 8$

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 3, 3, 3)$$

number of gaps

number of processes

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 3, 4, 6)$$

number of processes

the canonical cycloid

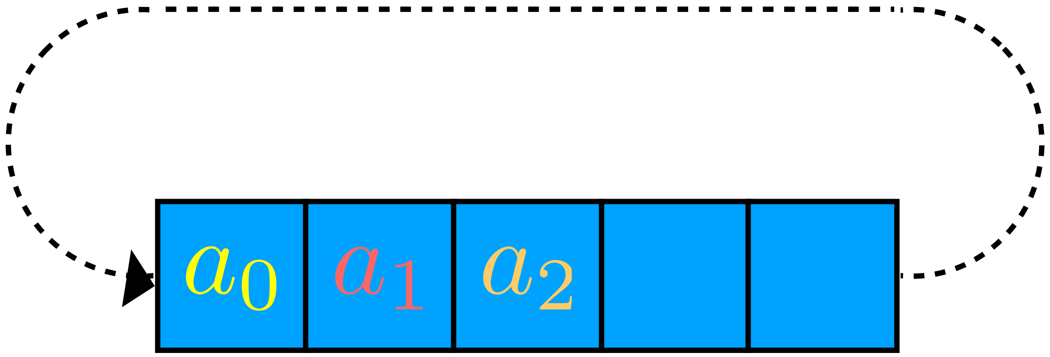
$\beta = \gamma = \delta$

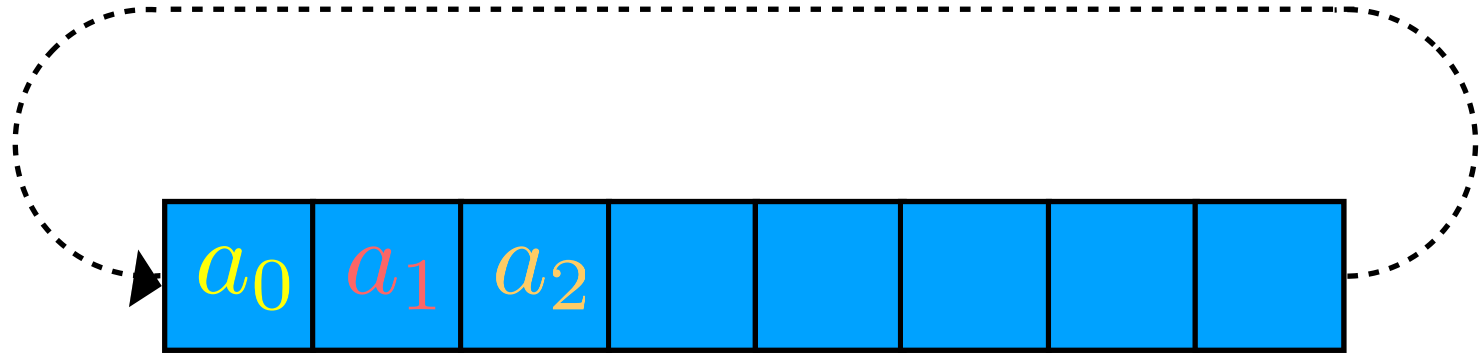
the regular cycloid

β devides δ

circular traffic queues of process length 5 and 8

circular queue of 3 items and 2 gaps





length of processes: $p = \frac{A}{\beta} = \frac{24}{3} = 8$

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 3, 3, 3)$$

number of gaps

number of processes

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 3, 4, 6)$$

number of processes

the canonical cycloid

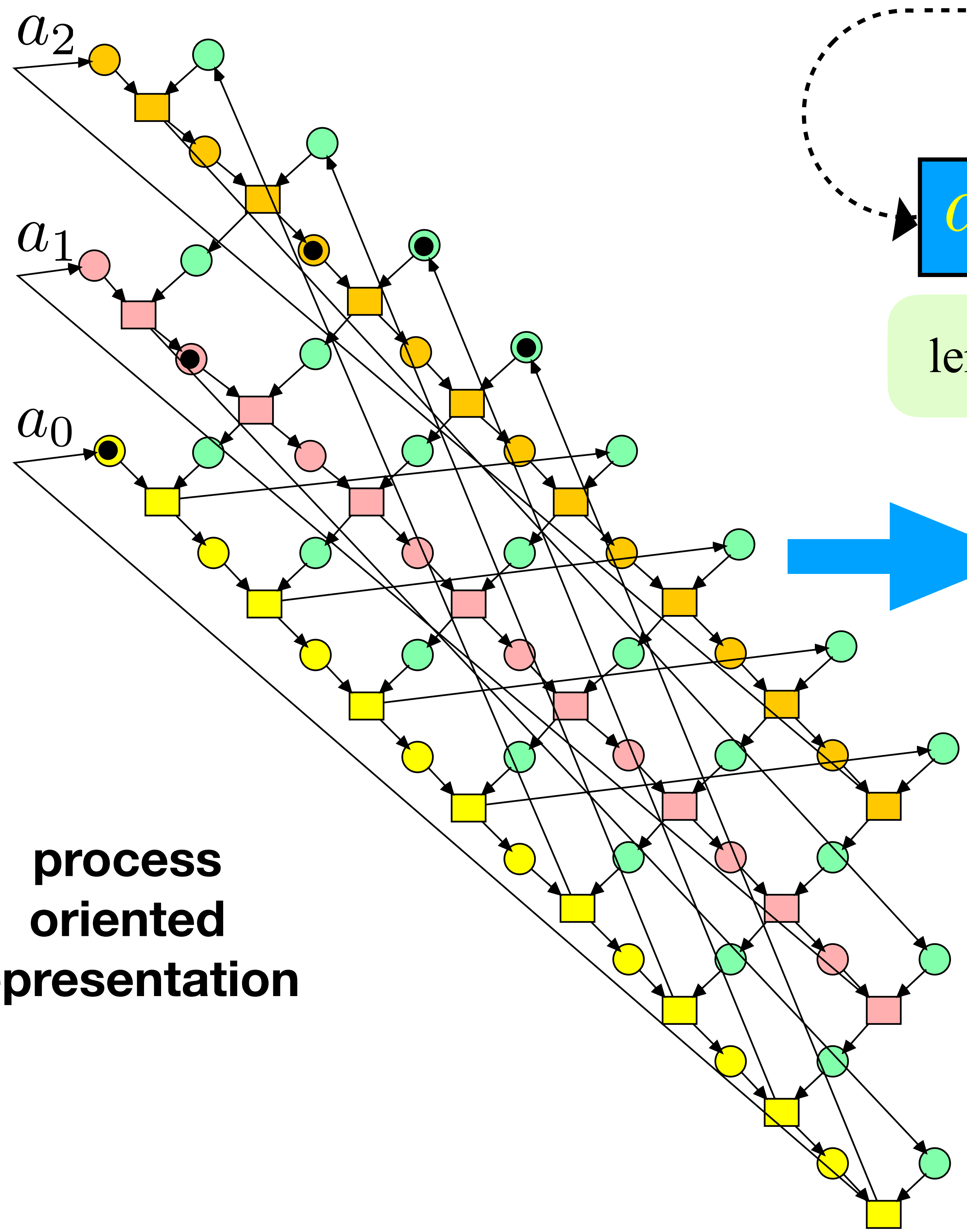
$$\beta = \gamma = \delta$$

the regular cycloid

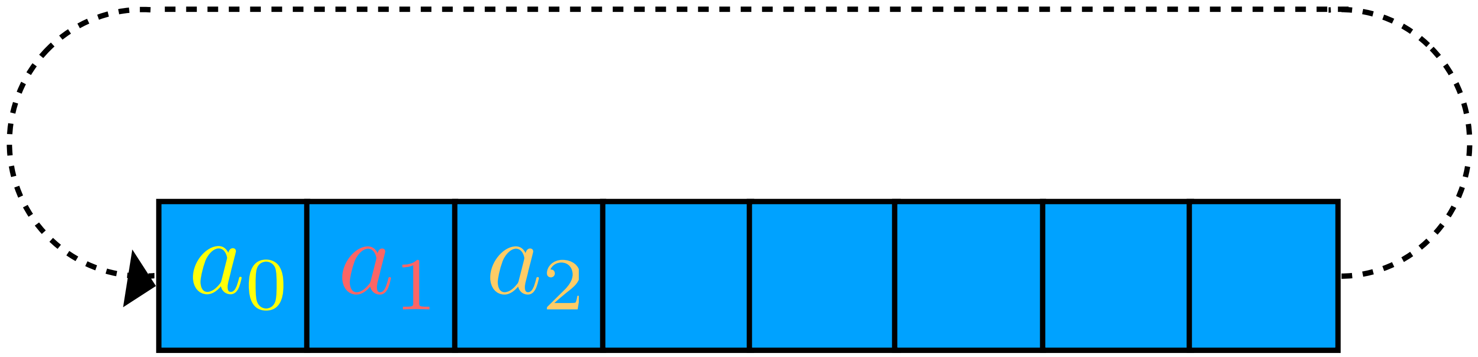
β devides δ

one token process structure

a circular traffic queue of process length 8



process oriented representation



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$$\mathcal{C}(\alpha, \beta, \gamma, \delta)$$
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number of processes

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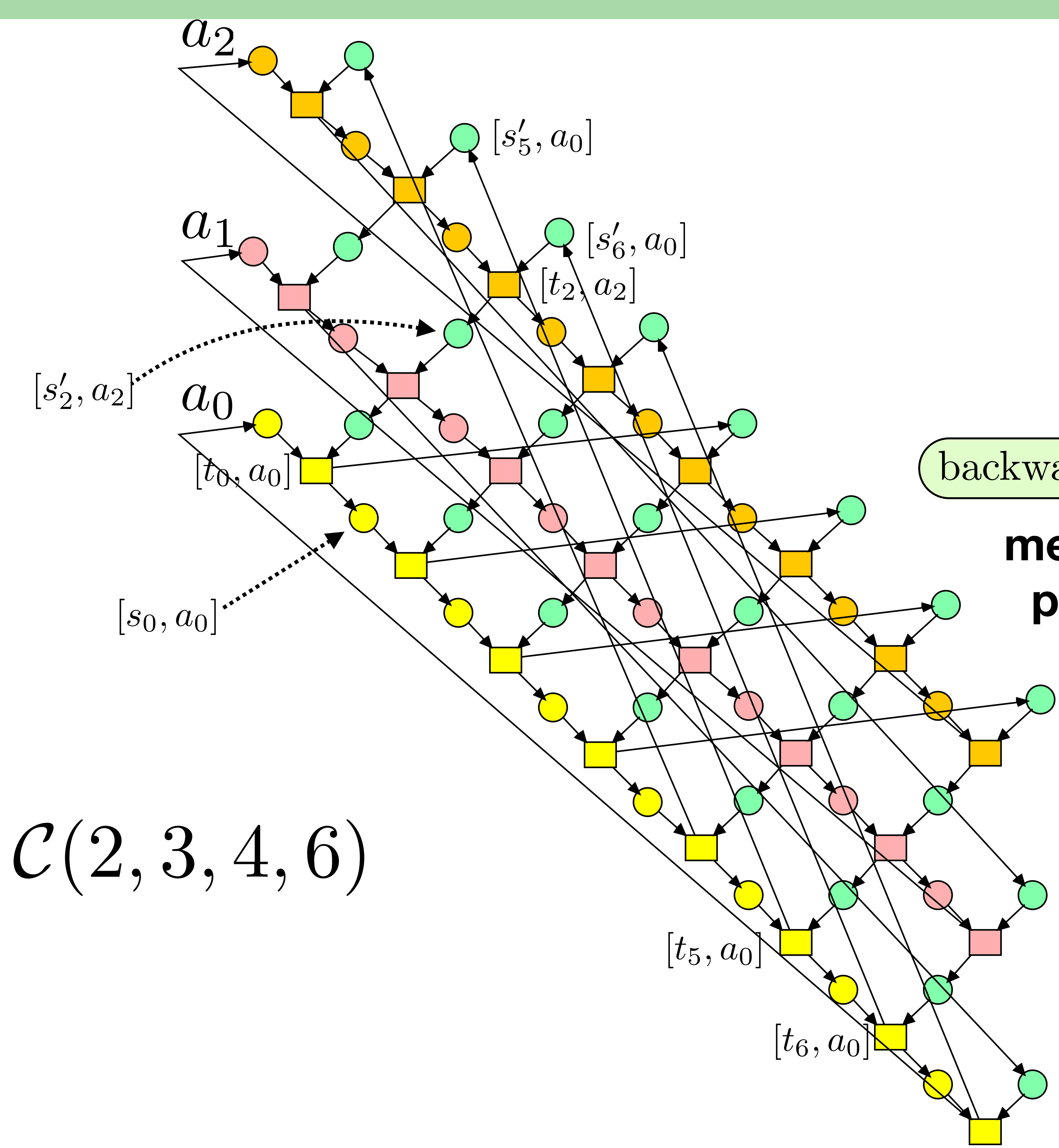
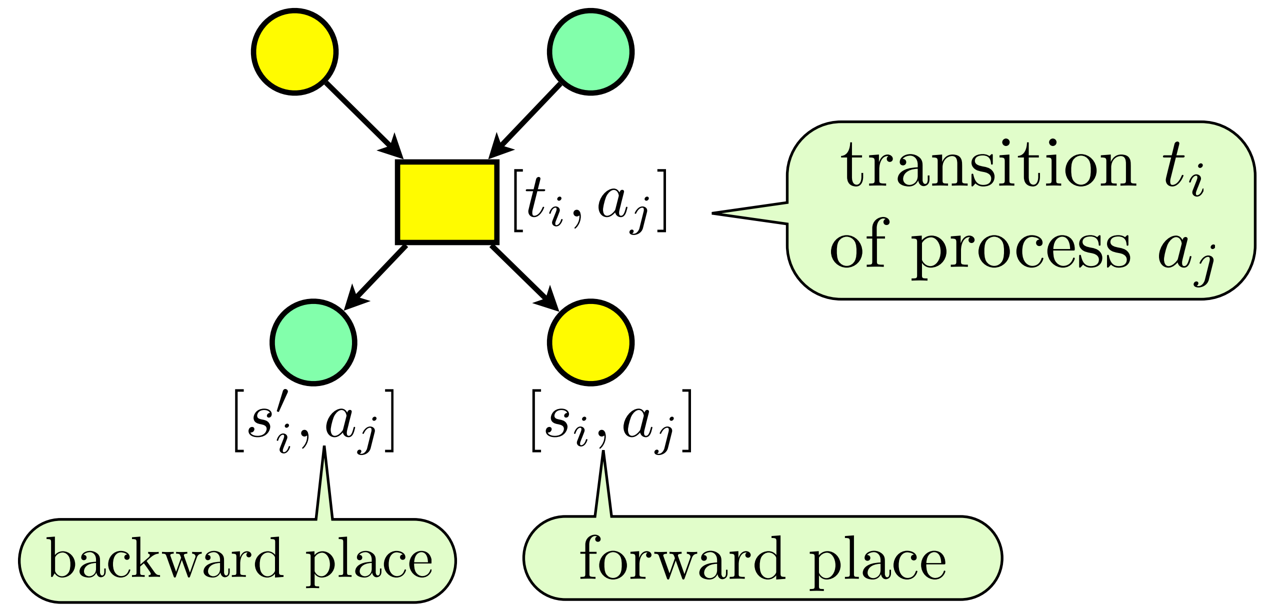
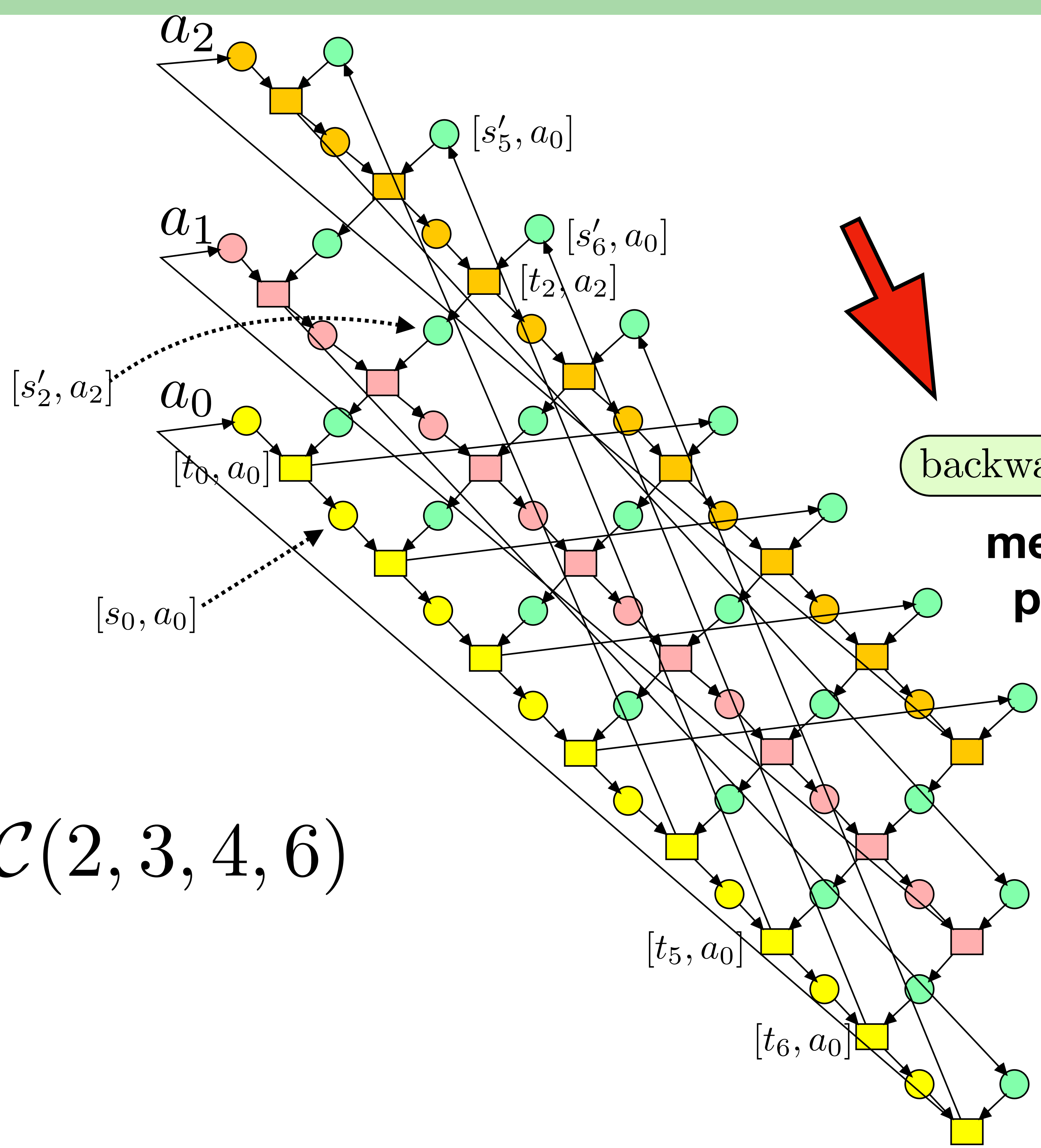
the backward folding



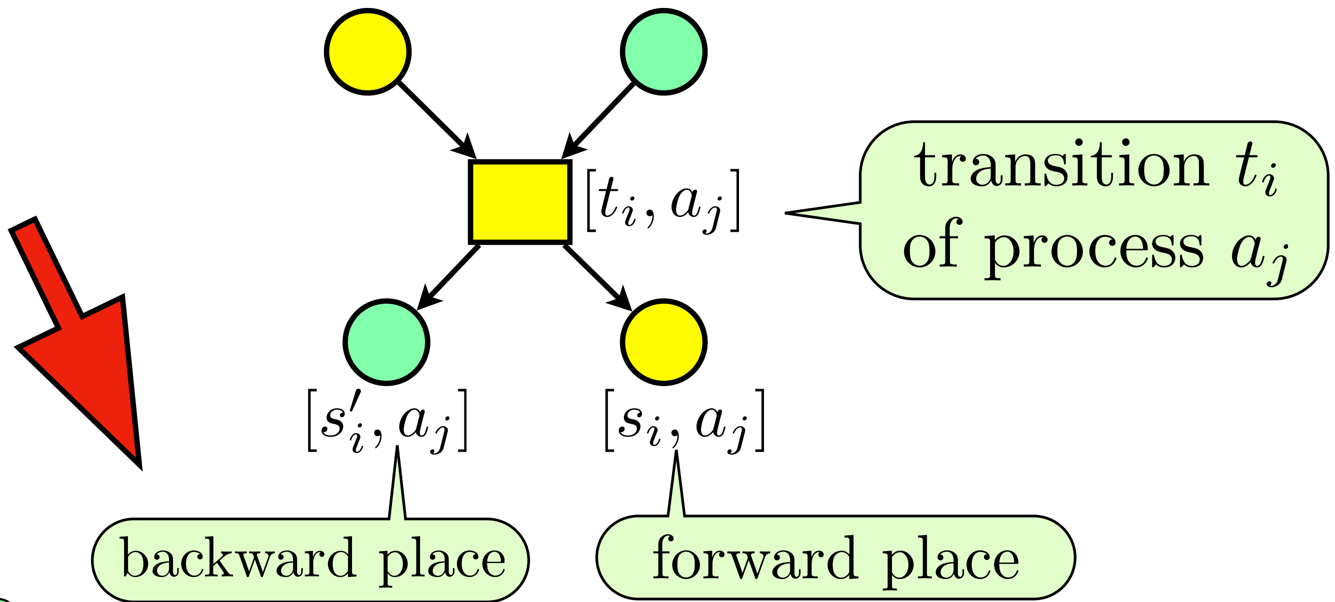


message places

process places



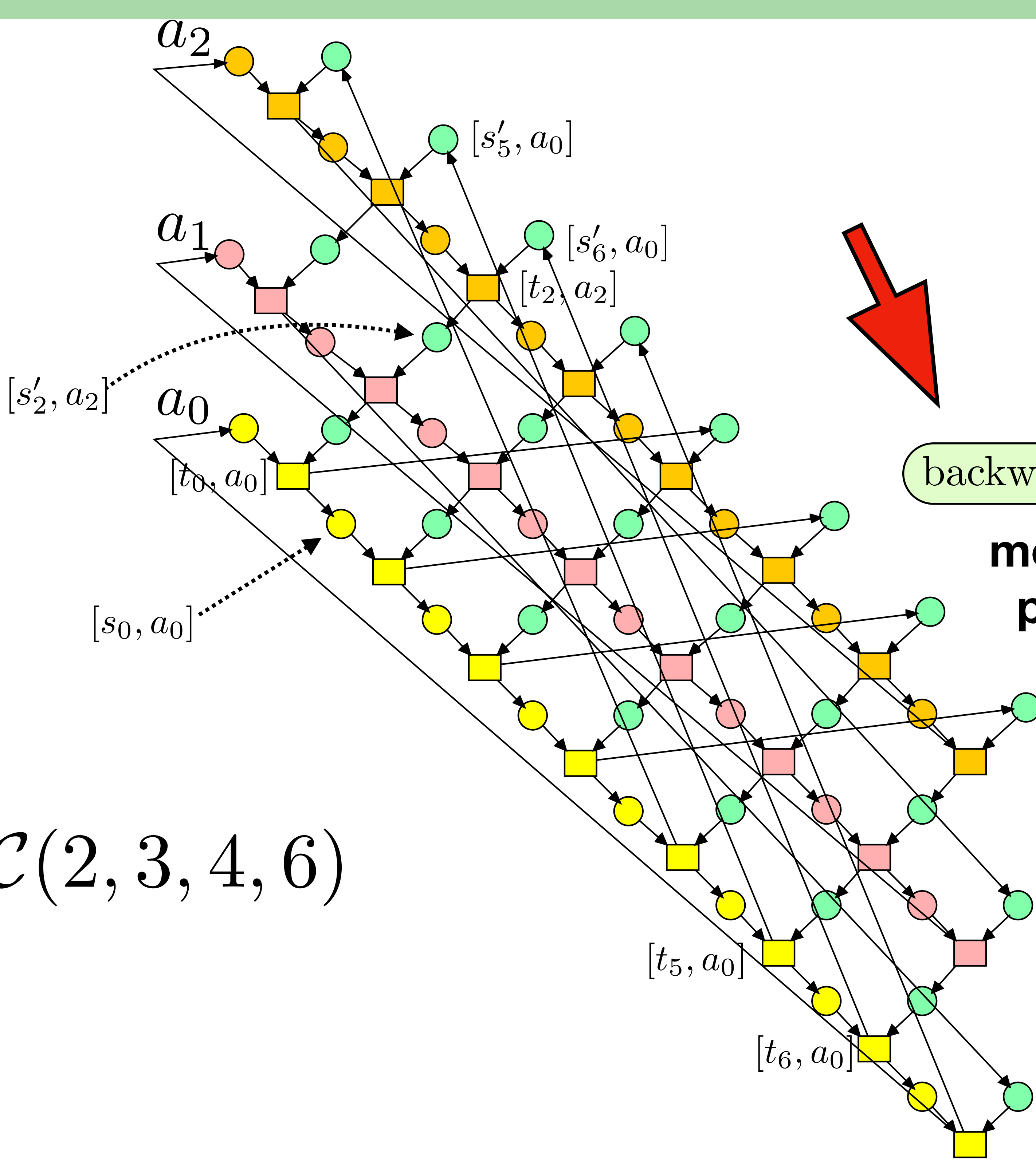
$\mathcal{C}(2,3,4,6)$



message places

process places

the backward folding



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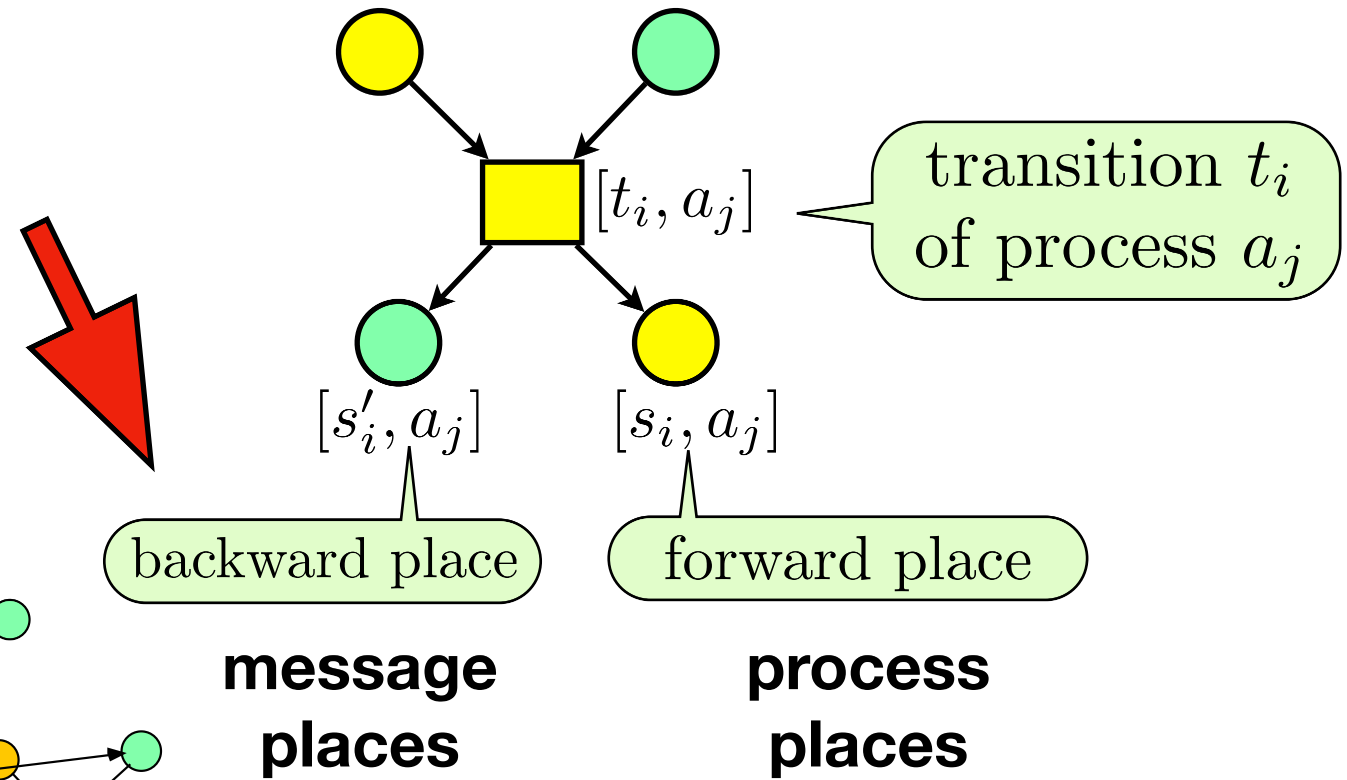
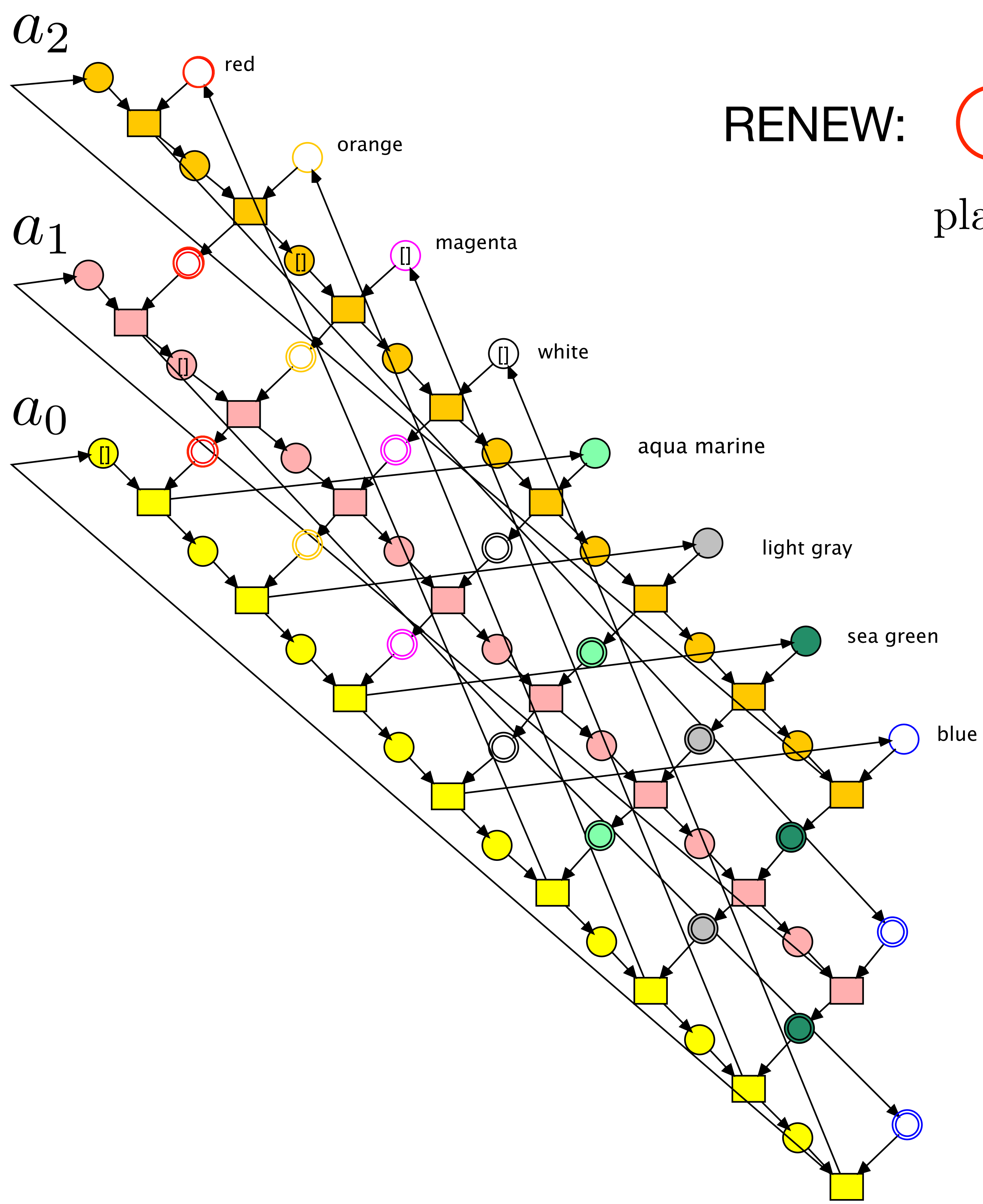


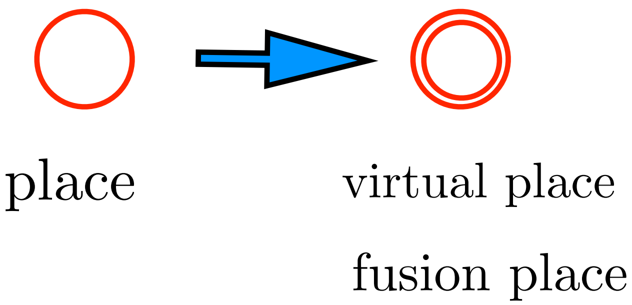
we define the backward folding

a folding of backward (= channel) places

the backward folding



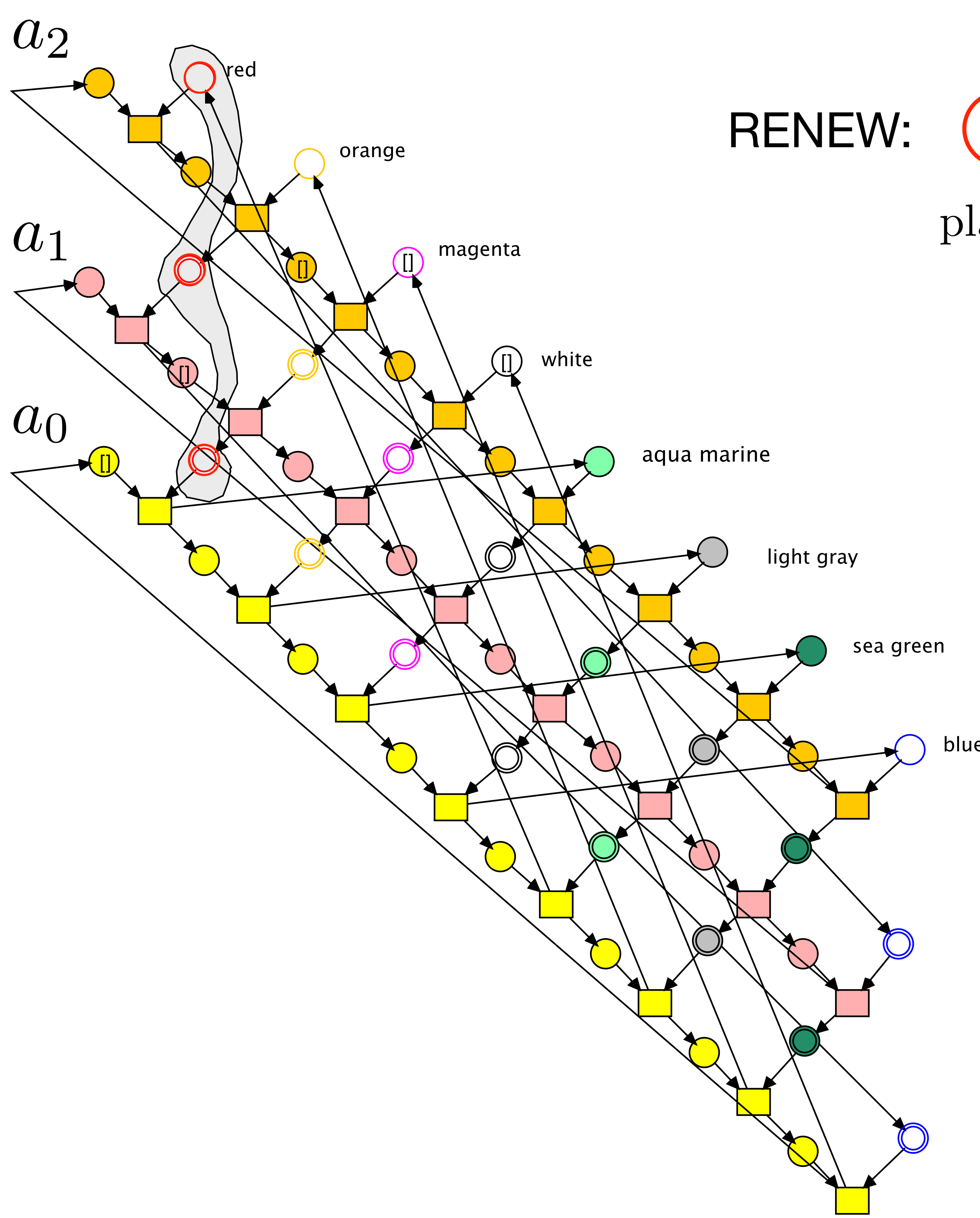
RENEW:



$\mathcal{C}(2,3,4,6)$

the backward folding

$\mathcal{C}(2,3,4,6)$



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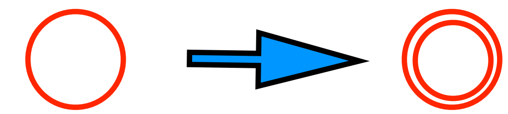
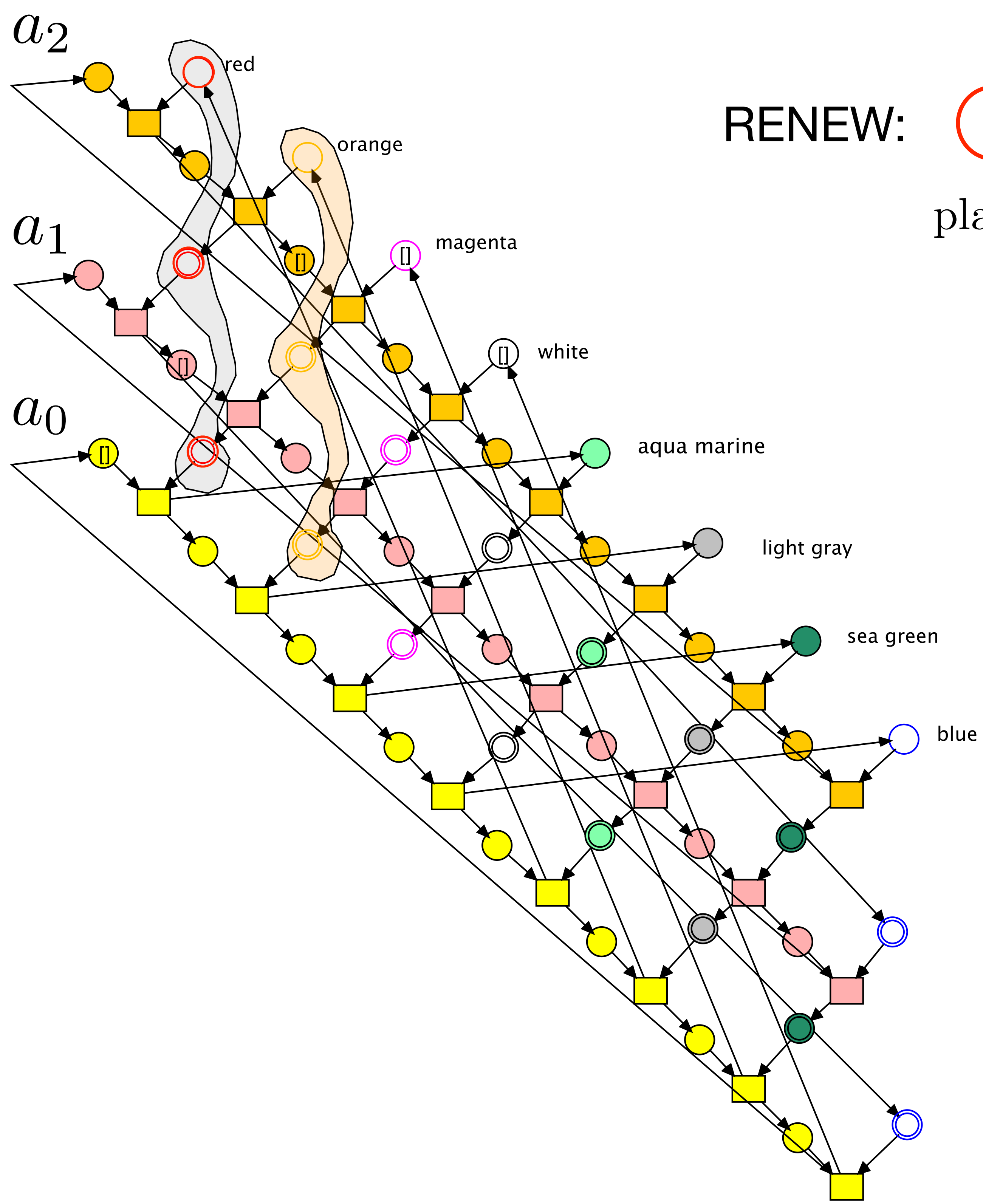


place

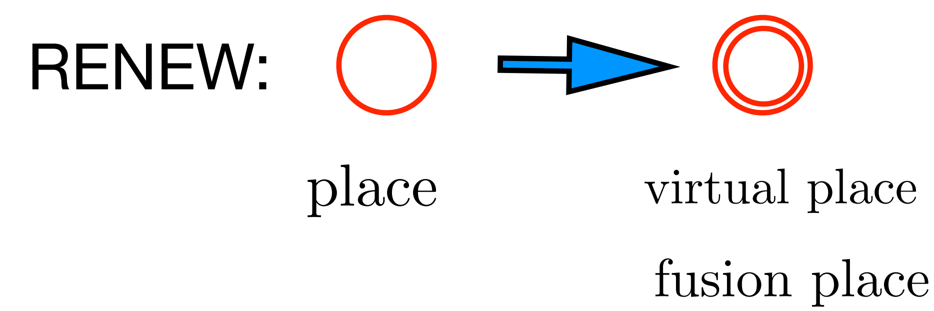
virtual place

fusion place

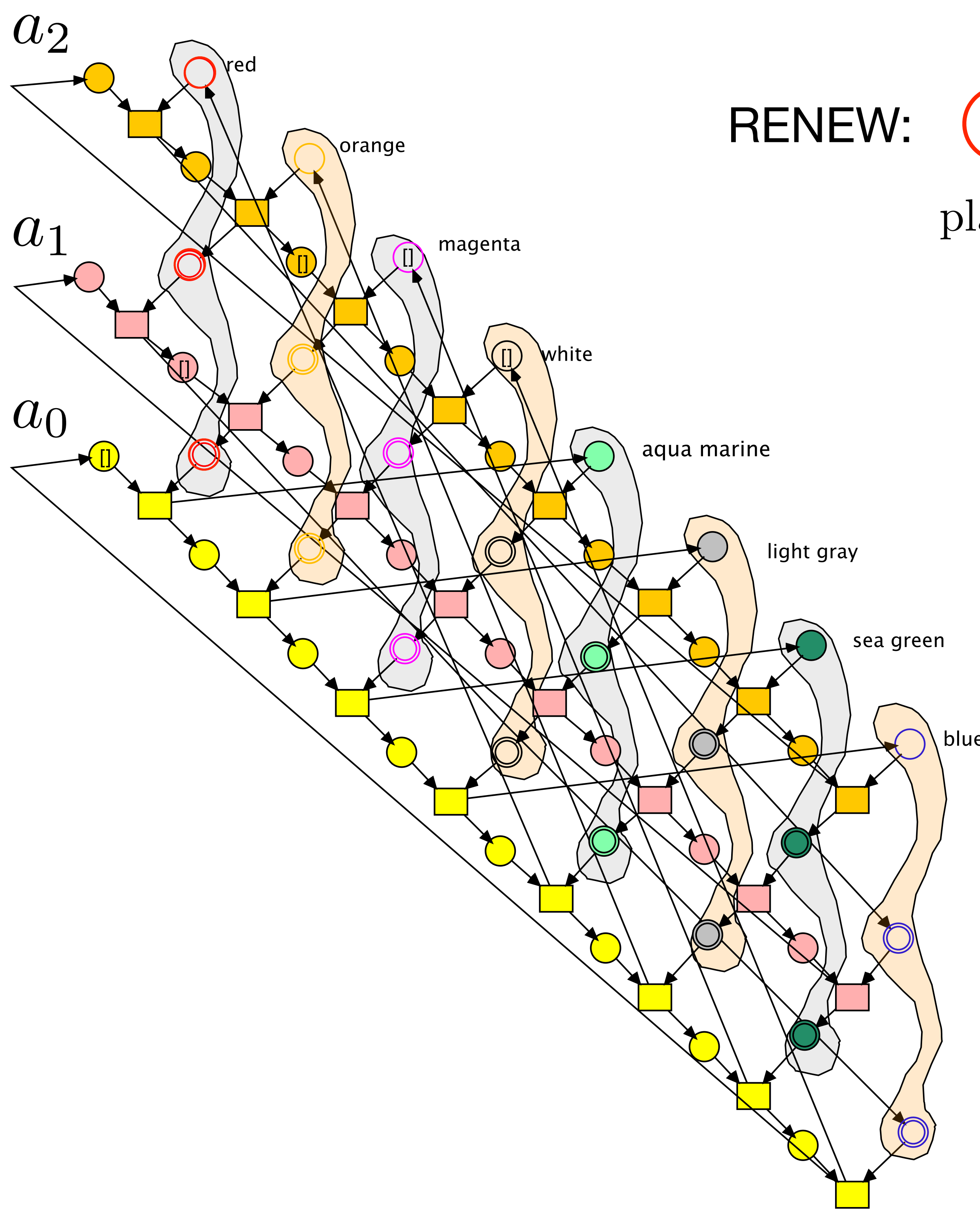
the backward folding

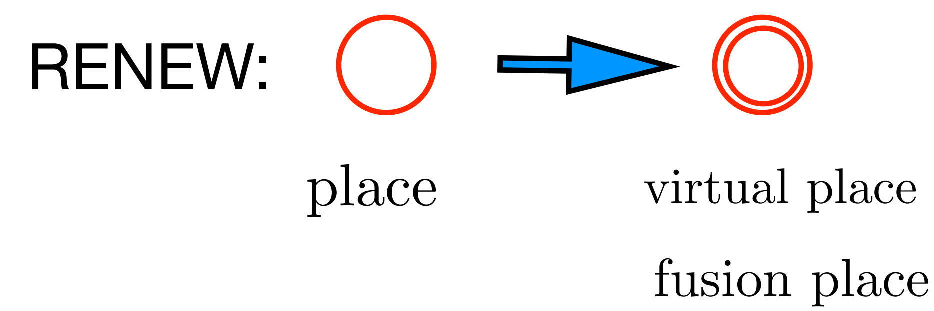


$\mathcal{C}(2,3,4,6)$

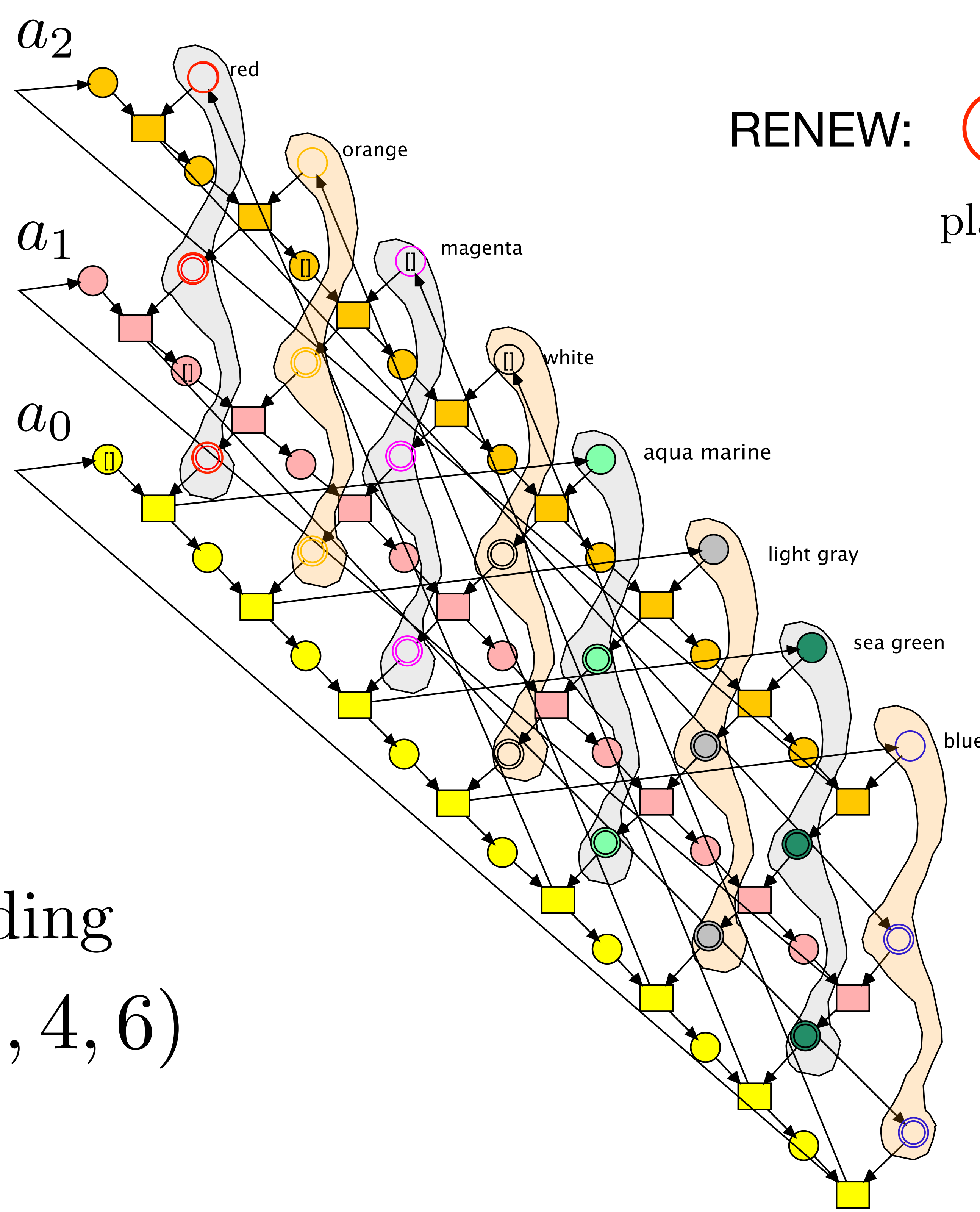


$\mathcal{C}(2,3,4,6)$

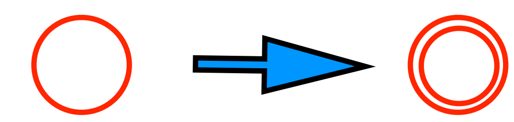




the backward folding



RENEW:

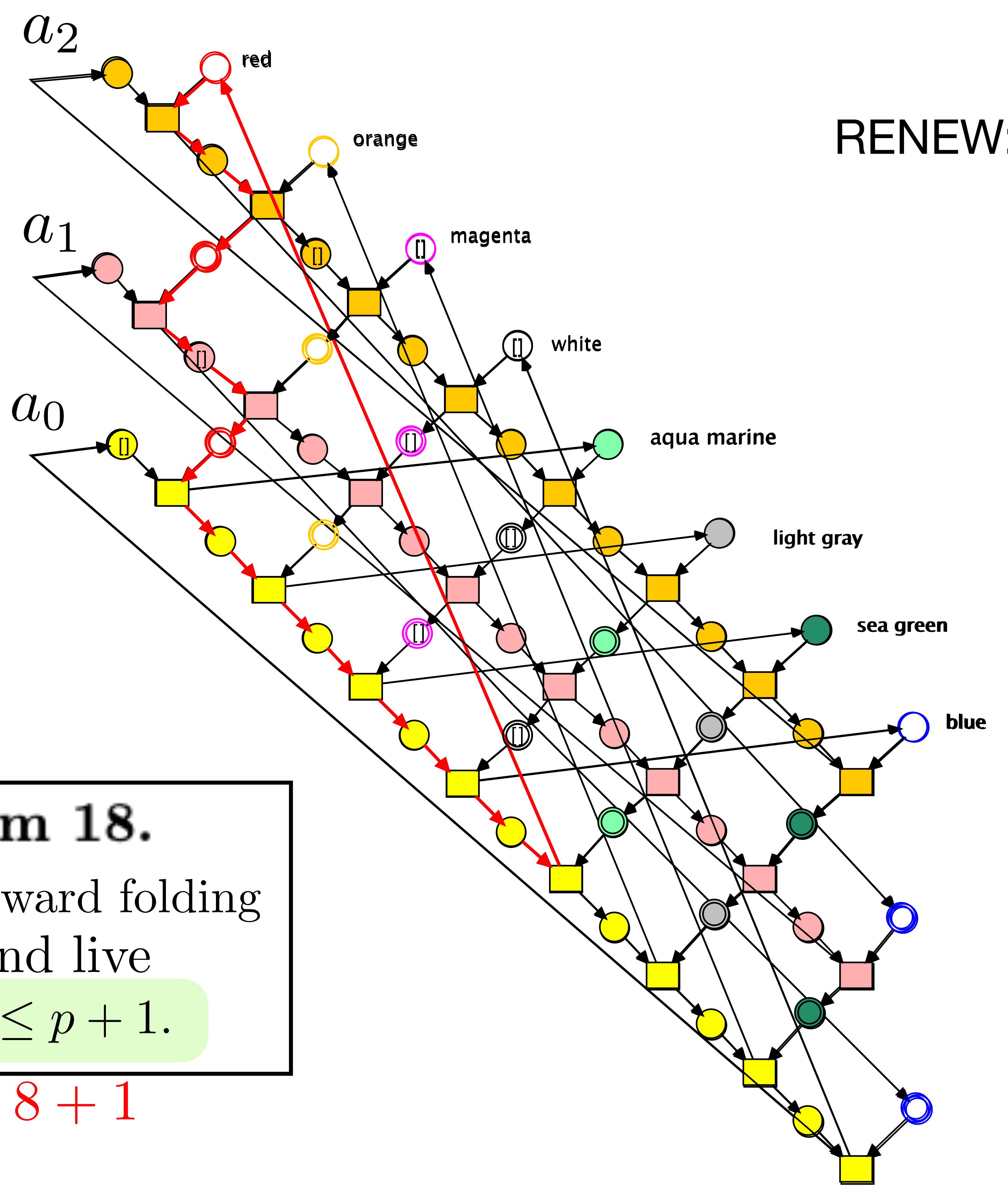


$$\mathcal{C}(2,3,4,6)$$

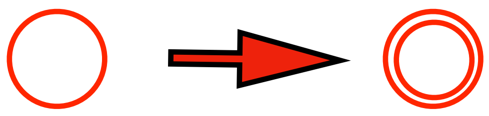
backward folding

$$\mathcal{C}_{bf}(2,3,4,6)$$

$\mathcal{C}_{bf}(2,3,4,6)$



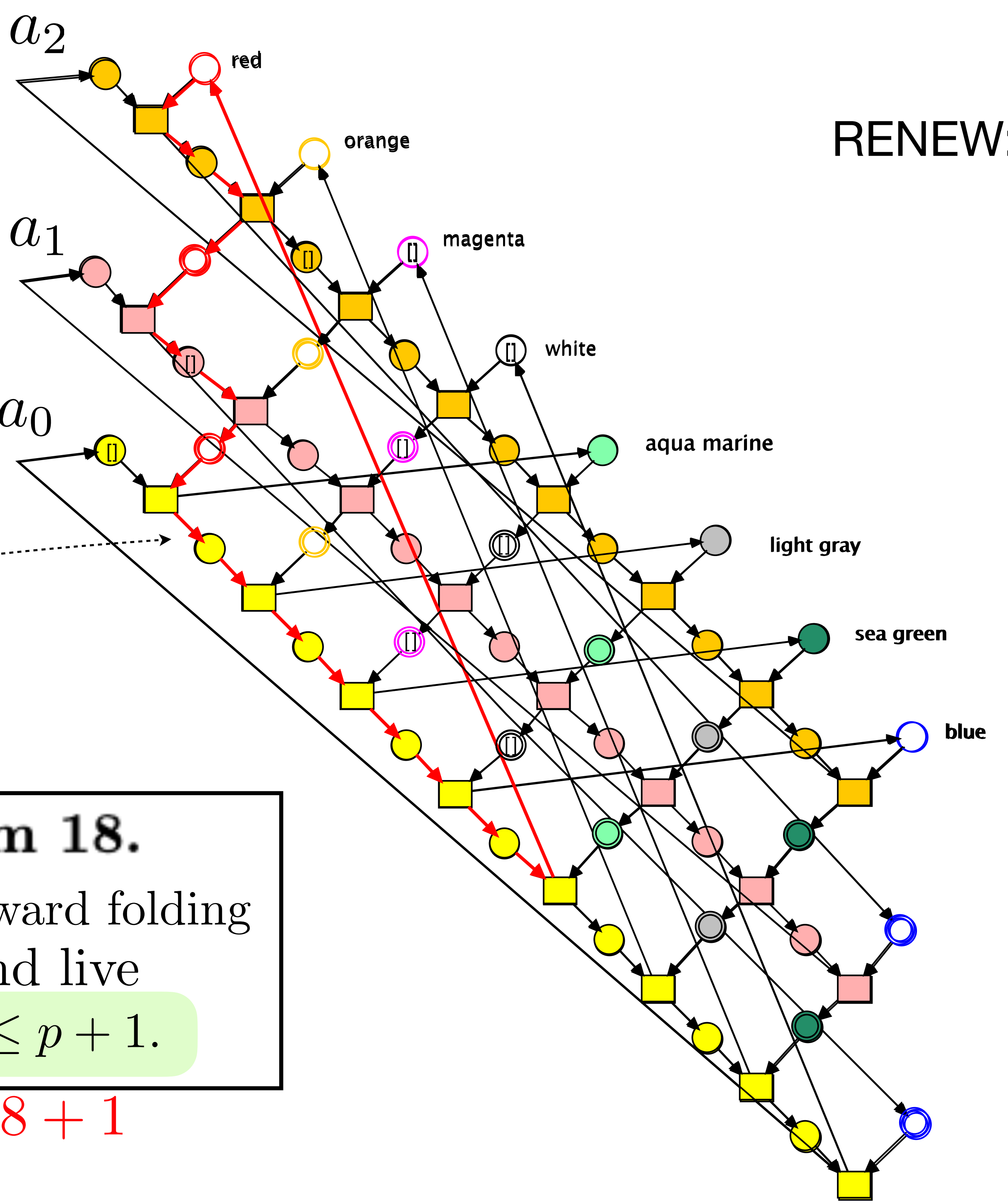
RENEW:



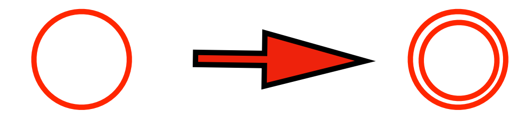
Theorem 18.
The backward folding is safe and live if $\alpha + \beta \leq p + 1$.

$2 + 3 \leq 8 + 1$

$\mathcal{C}_{bf}(2,3,4,6)$



RENEW:



place

virtual place

fusion place

a P-invariant

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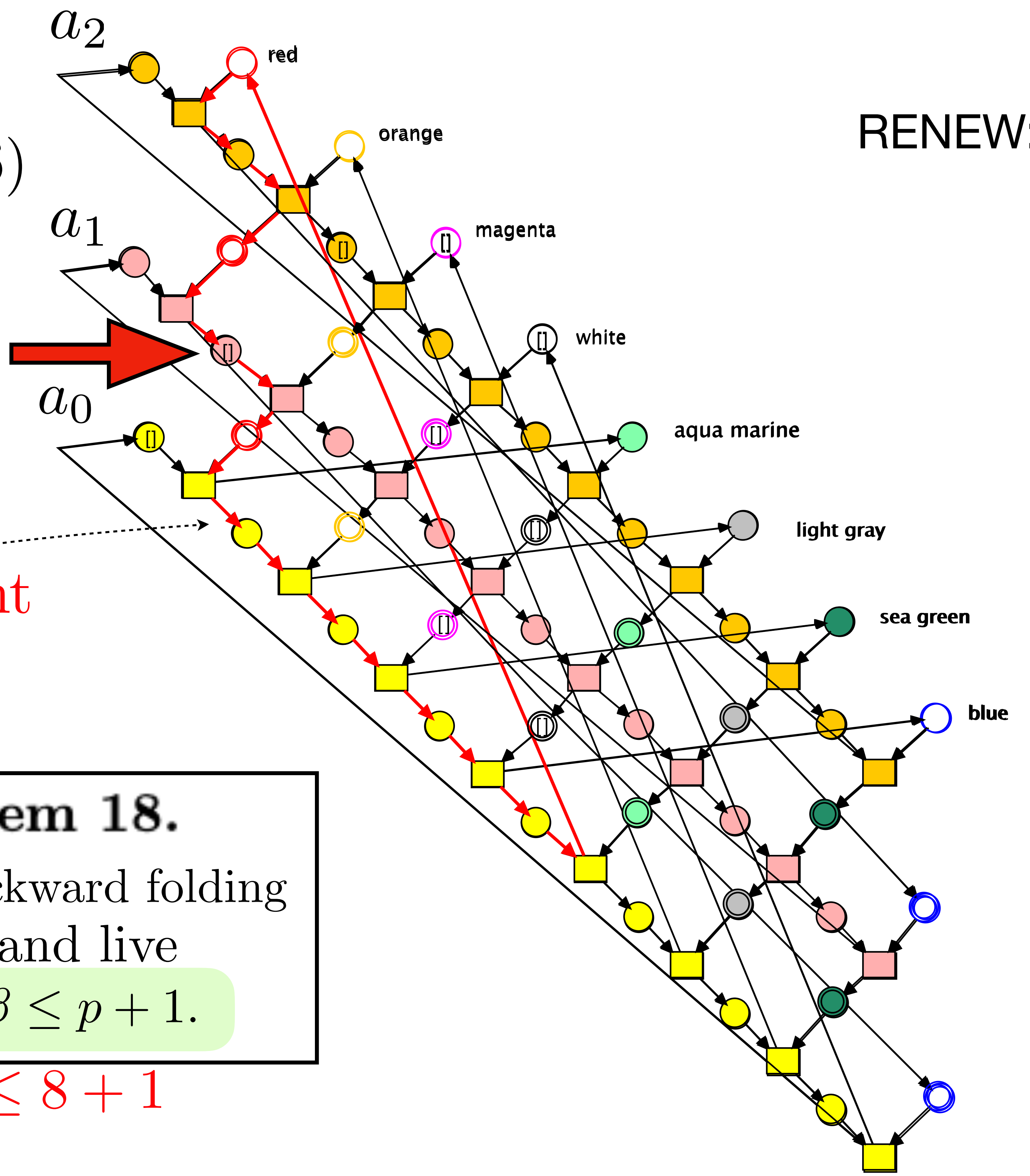
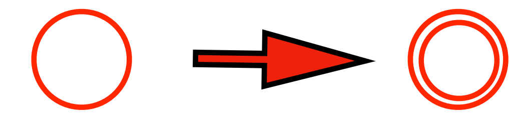
$\mathcal{C}_{bf}(2,3,4,6)$



a P-invariant

RENEW:



place

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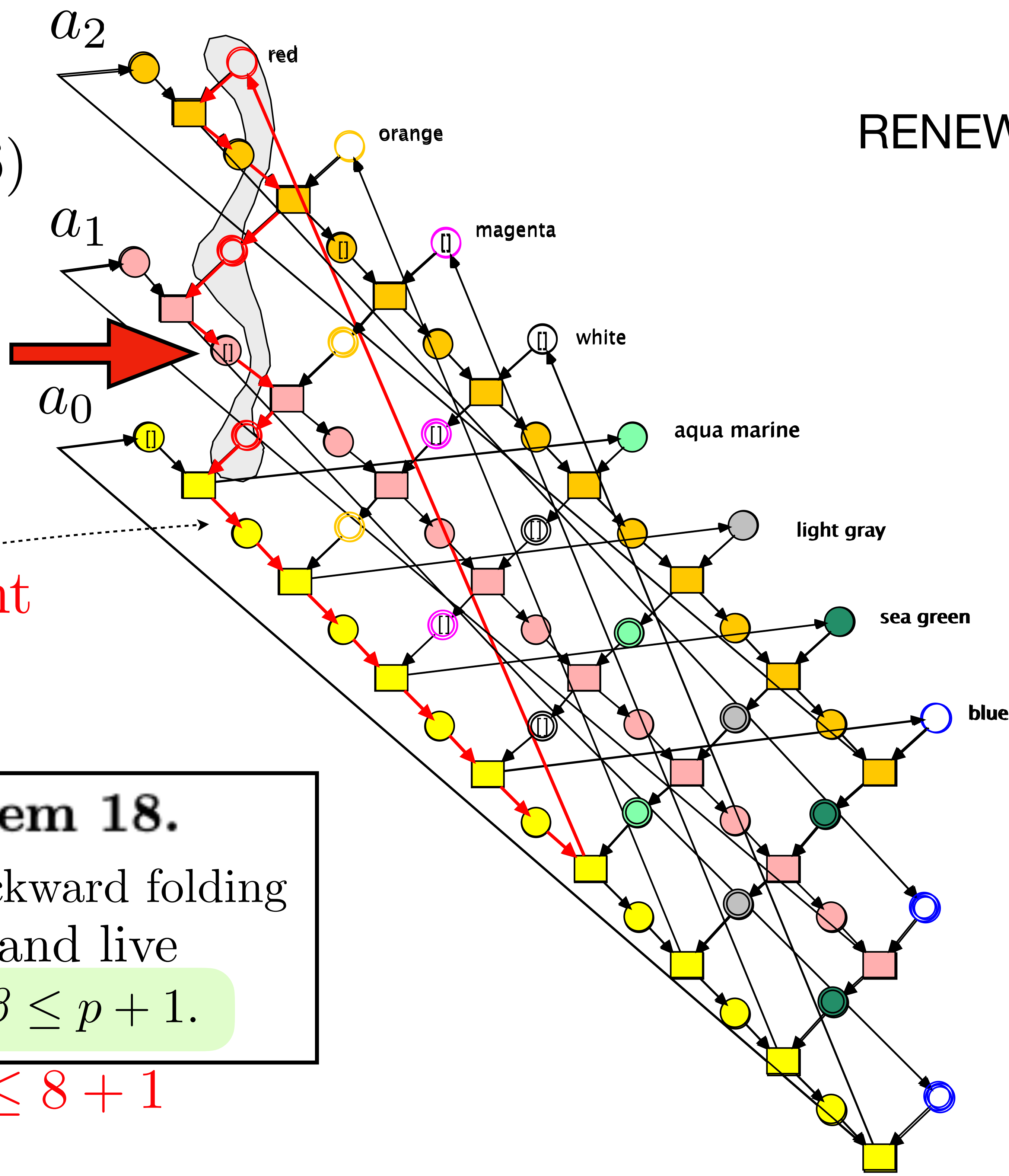

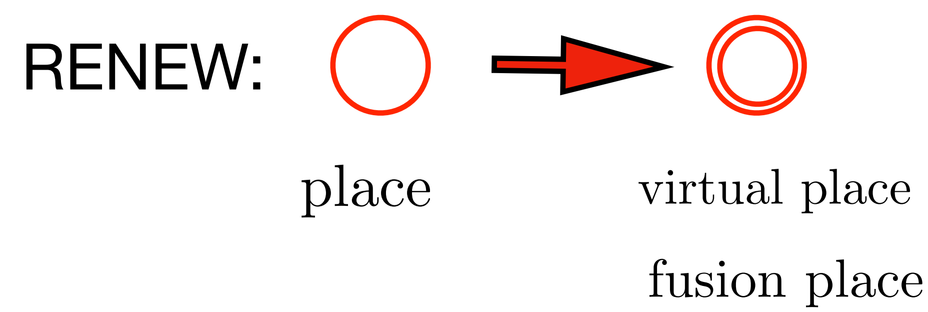

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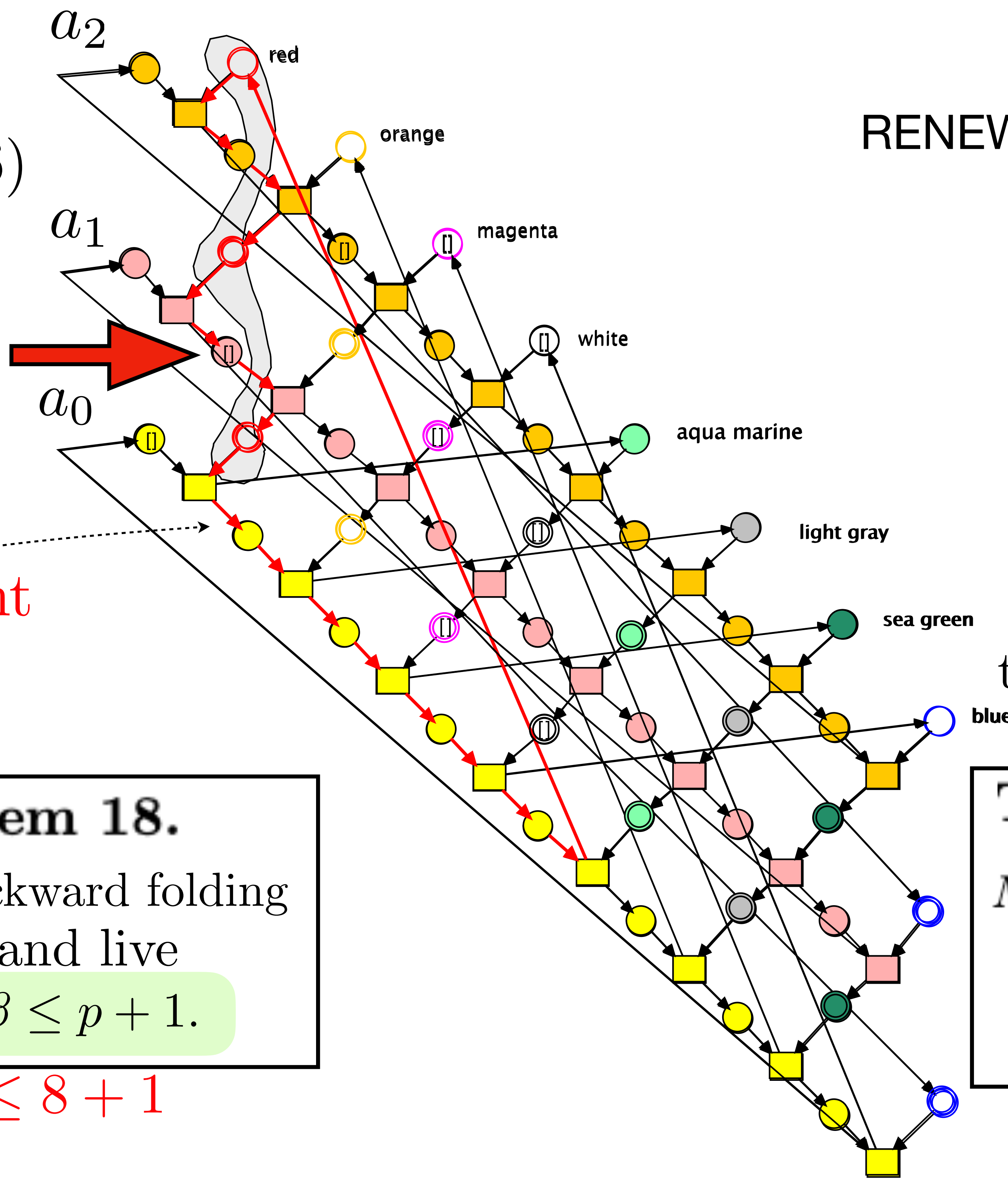
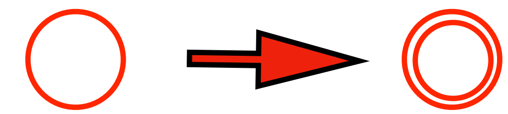


a P-invariant

RENEW:



place

virtual place

fusion place

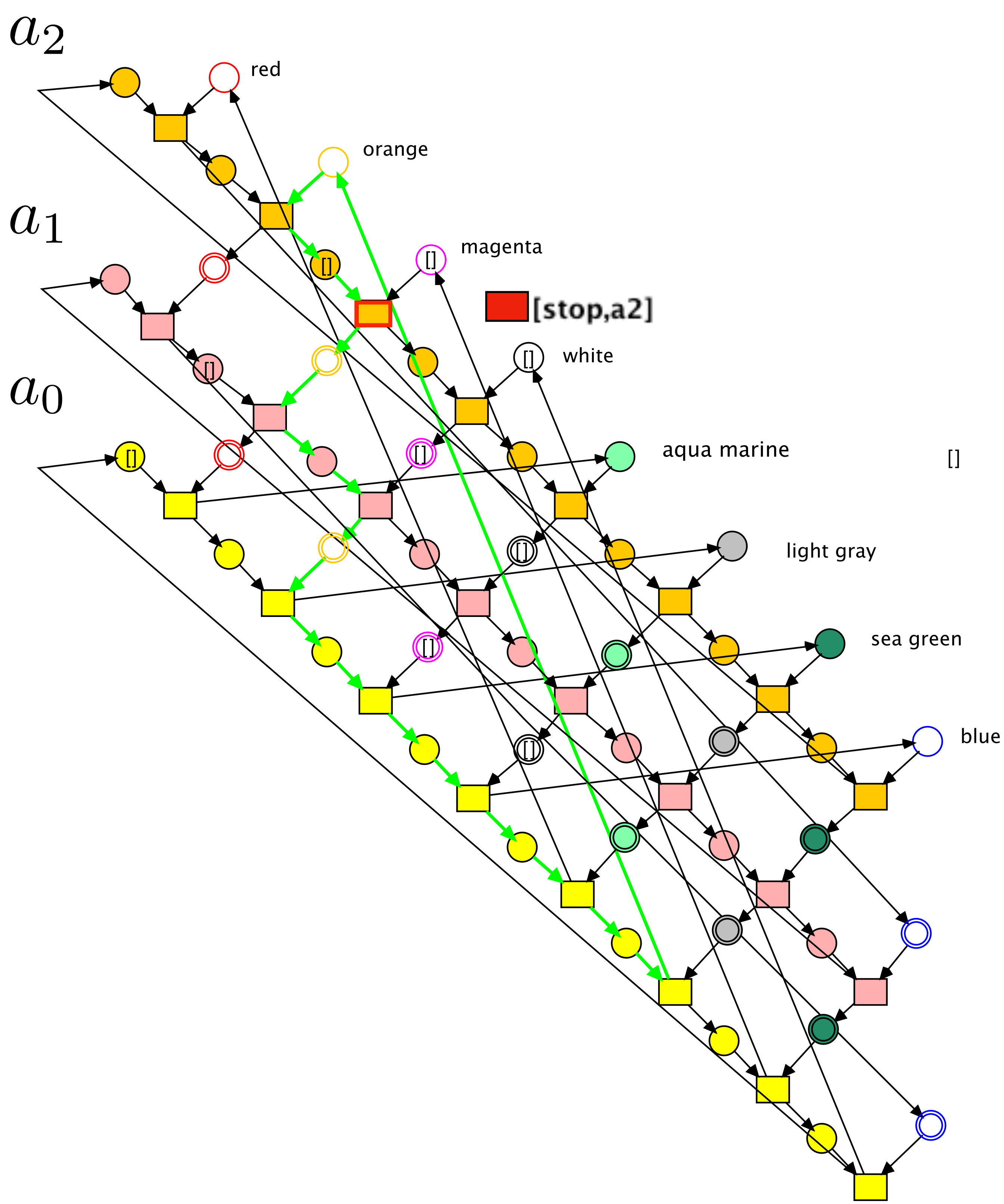
the behaviour is unchanged

Theorem 18.
The backward folding is safe and live if $\alpha + \beta \leq p + 1$.

$2 + 3 \leq 8 + 1$

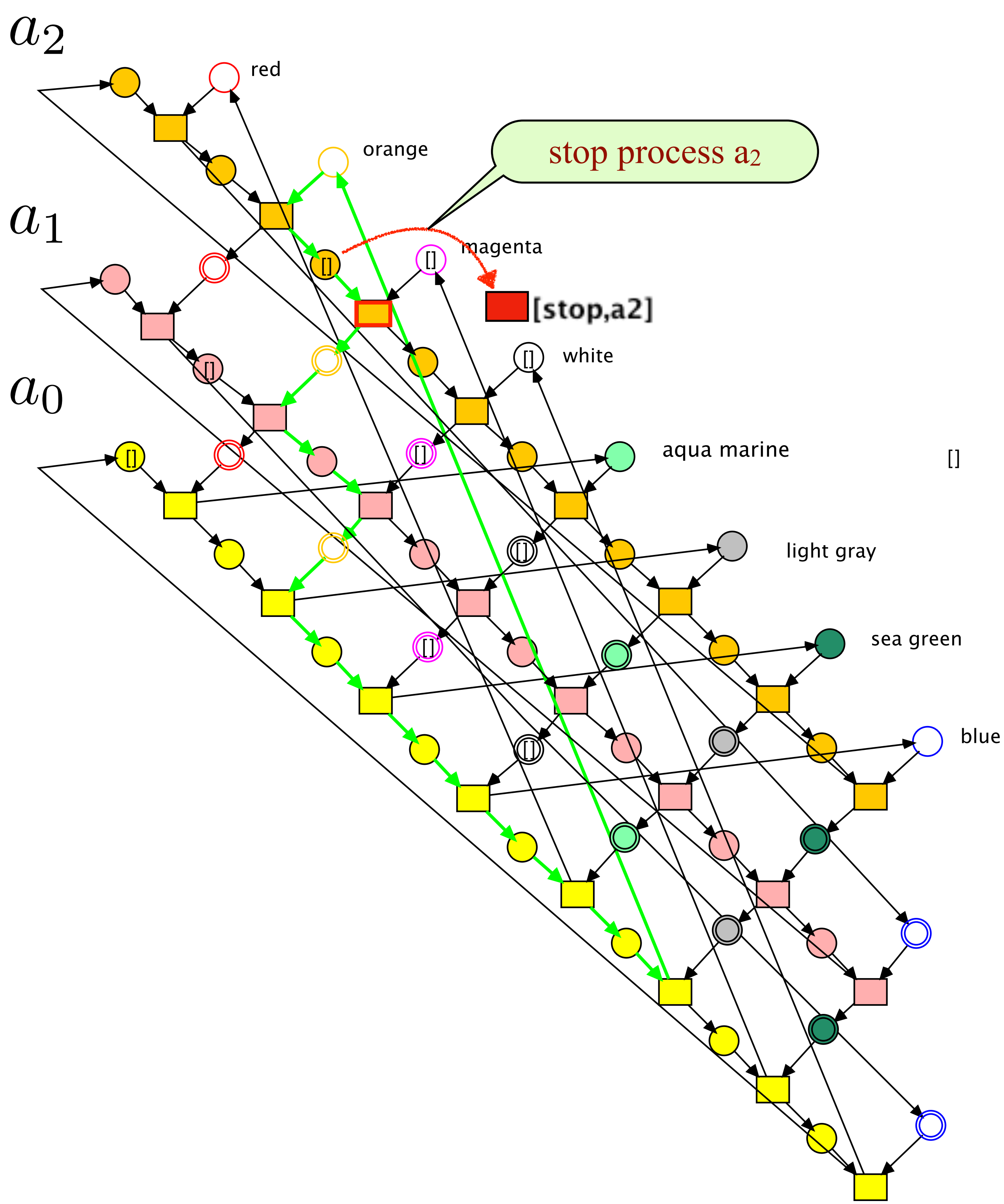
Theorem 19.
$$M \xrightarrow{t} M' \;\Leftrightarrow\; [\![M]\!]_D \xrightarrow[bf(D)]{t} [\![M']\!]_D$$
if $\alpha + \beta \leq p + 1$.

$\mathcal{C}_{bf}(2, 3, 4, 6)$



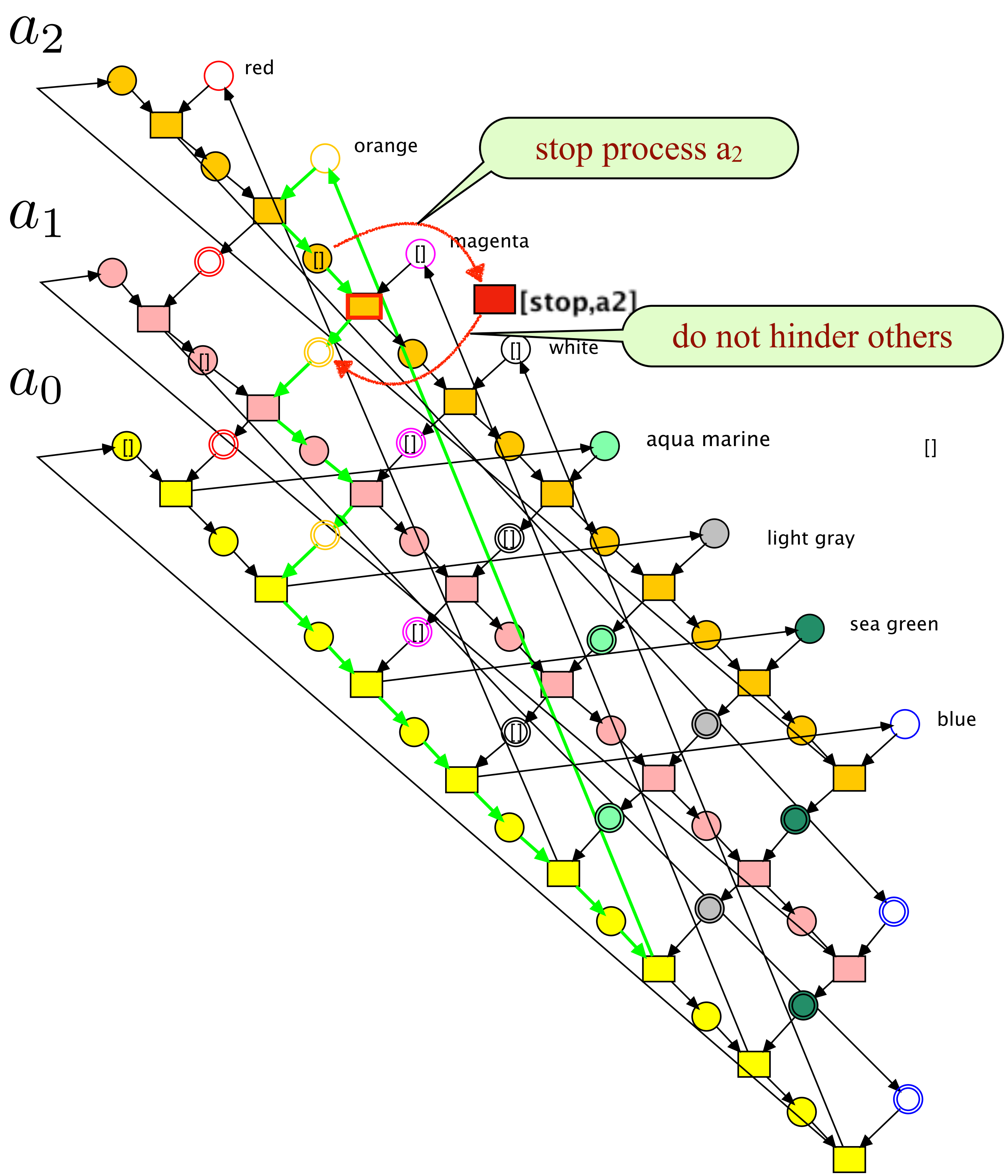
stopping process a_2

$\mathcal{C}_{bf}(2,3,4,6)$



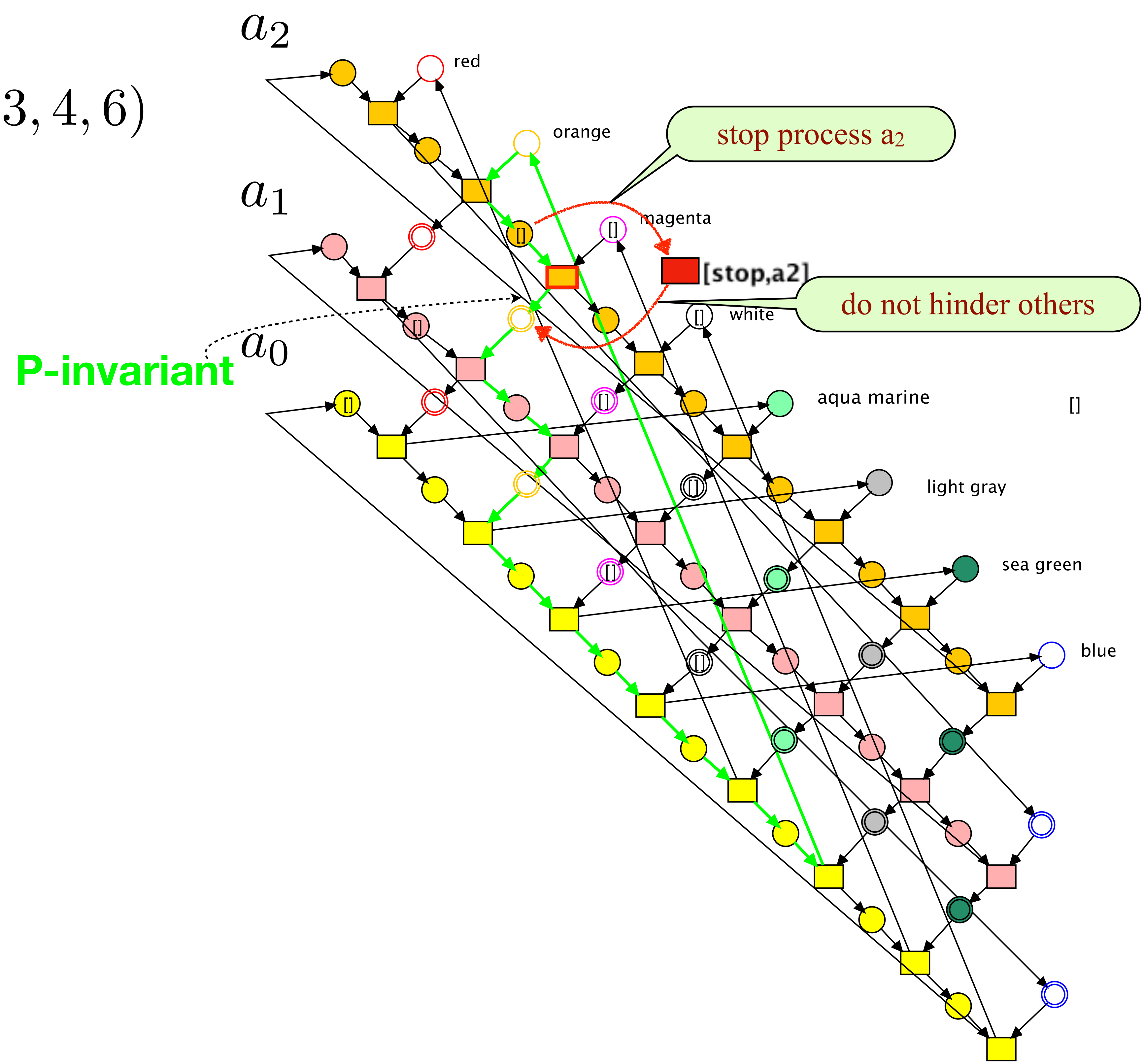
stopping process a_2

$\mathcal{C}_{bf}(2,3,4,6)$

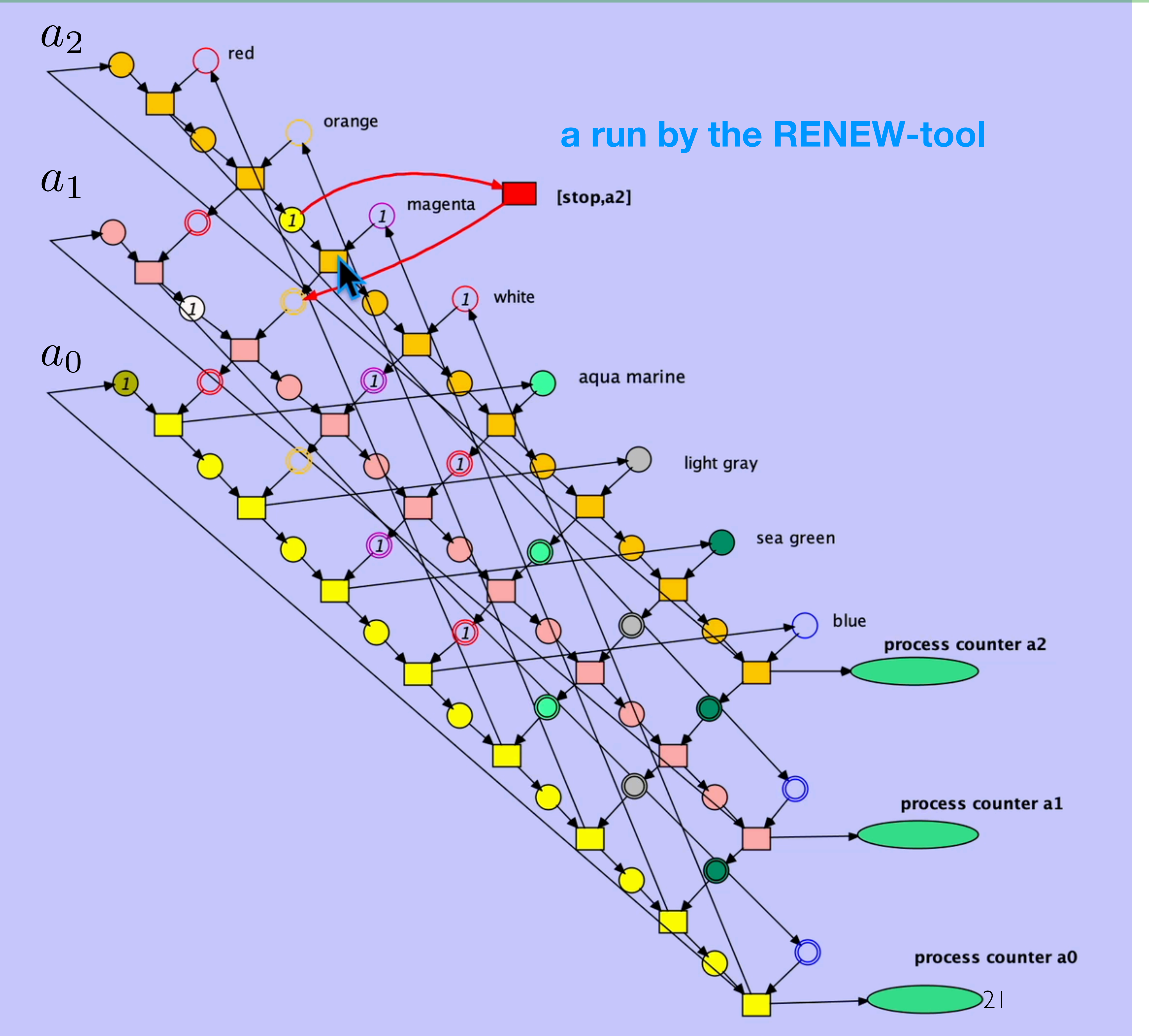


stopping process a_2

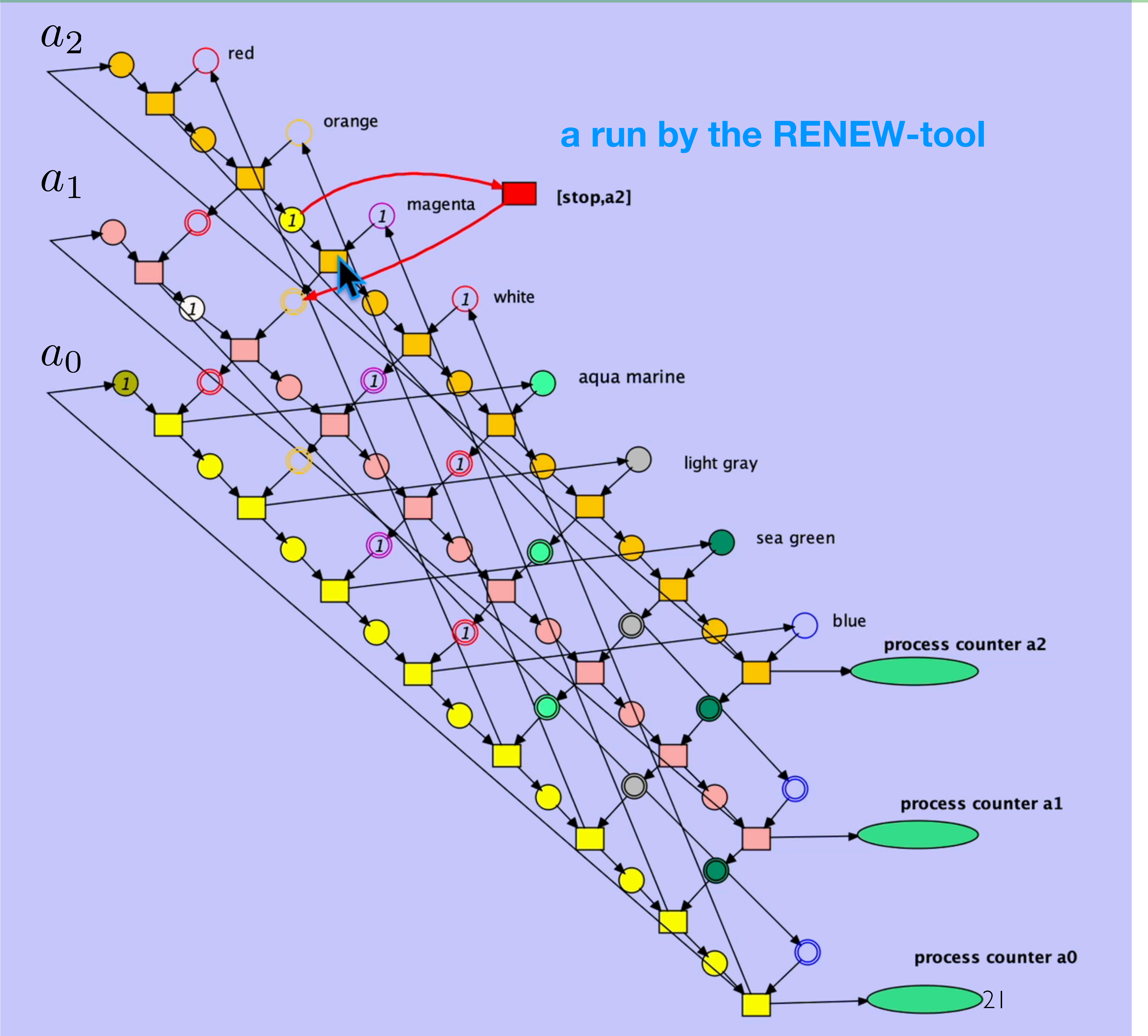
$\mathcal{C}_{bf}(2,3,4,6)$



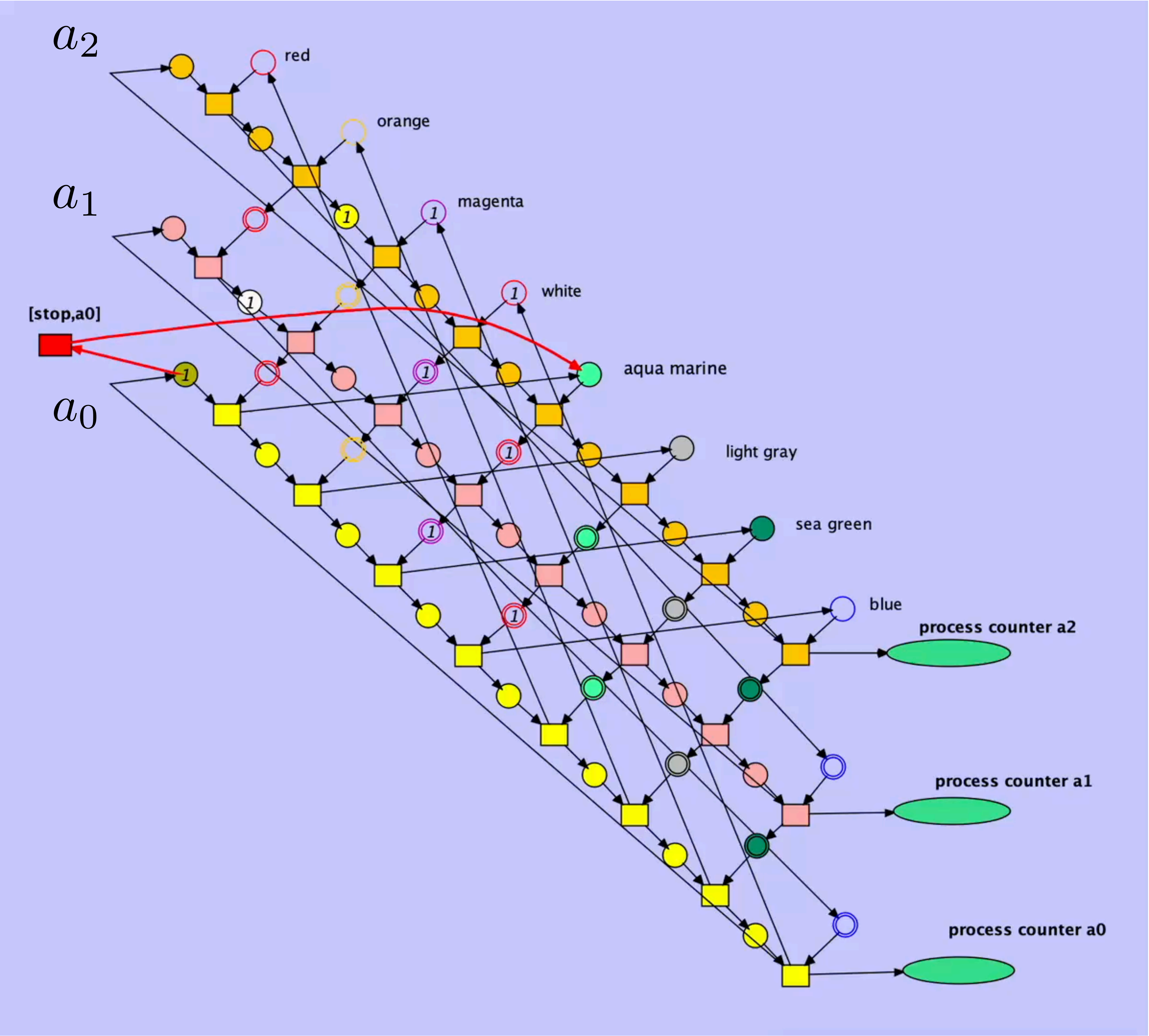
$\mathcal{C}_{bf}(2,3,4,6)$



$$\mathcal{C}_{bf}(2,3,4,6)$$

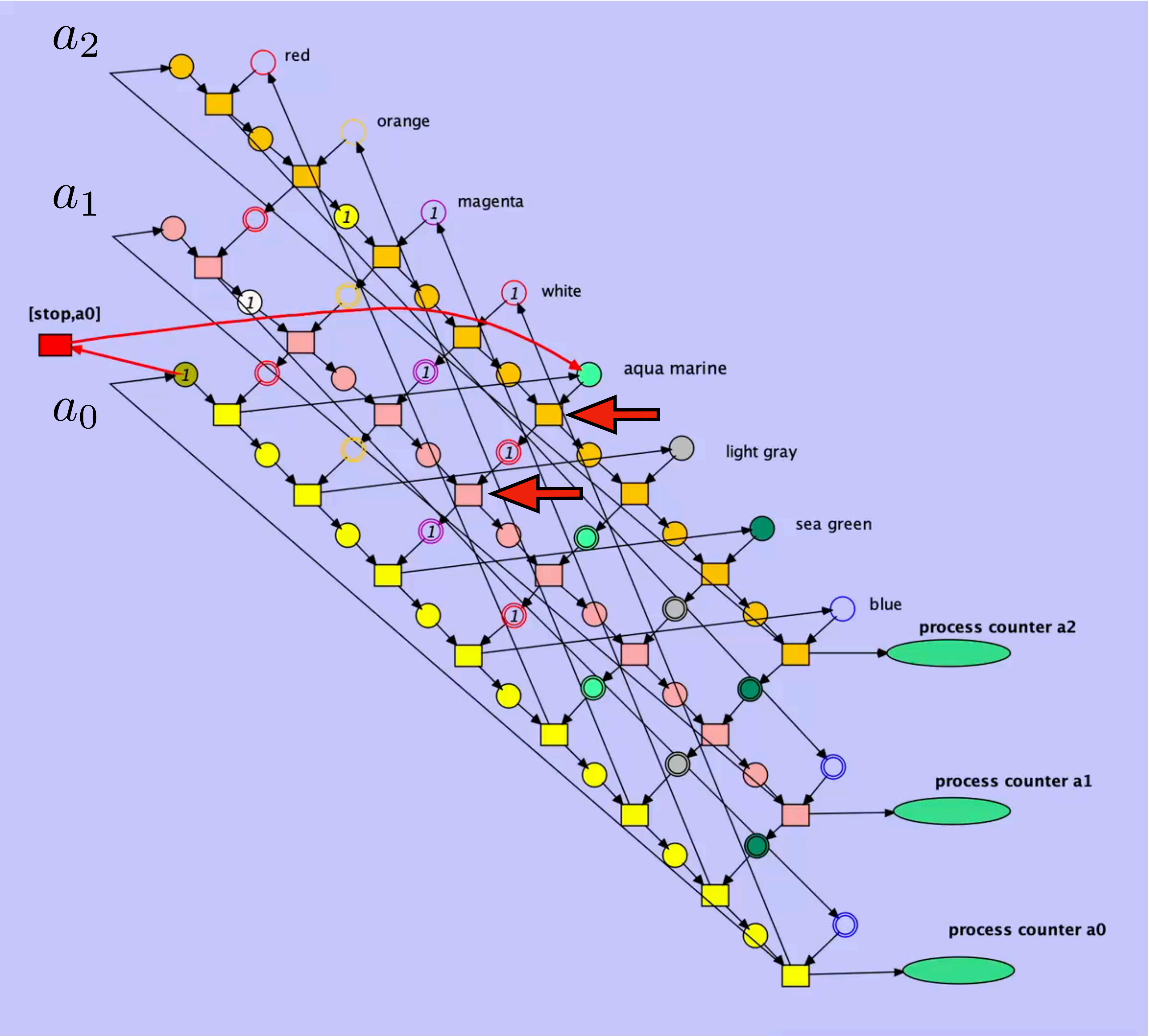


$\mathcal{C}_{bf}(2,3,4,6)$



$\mathcal{C}_{bf}(2,3,4,6)$

A deadlock occurs !



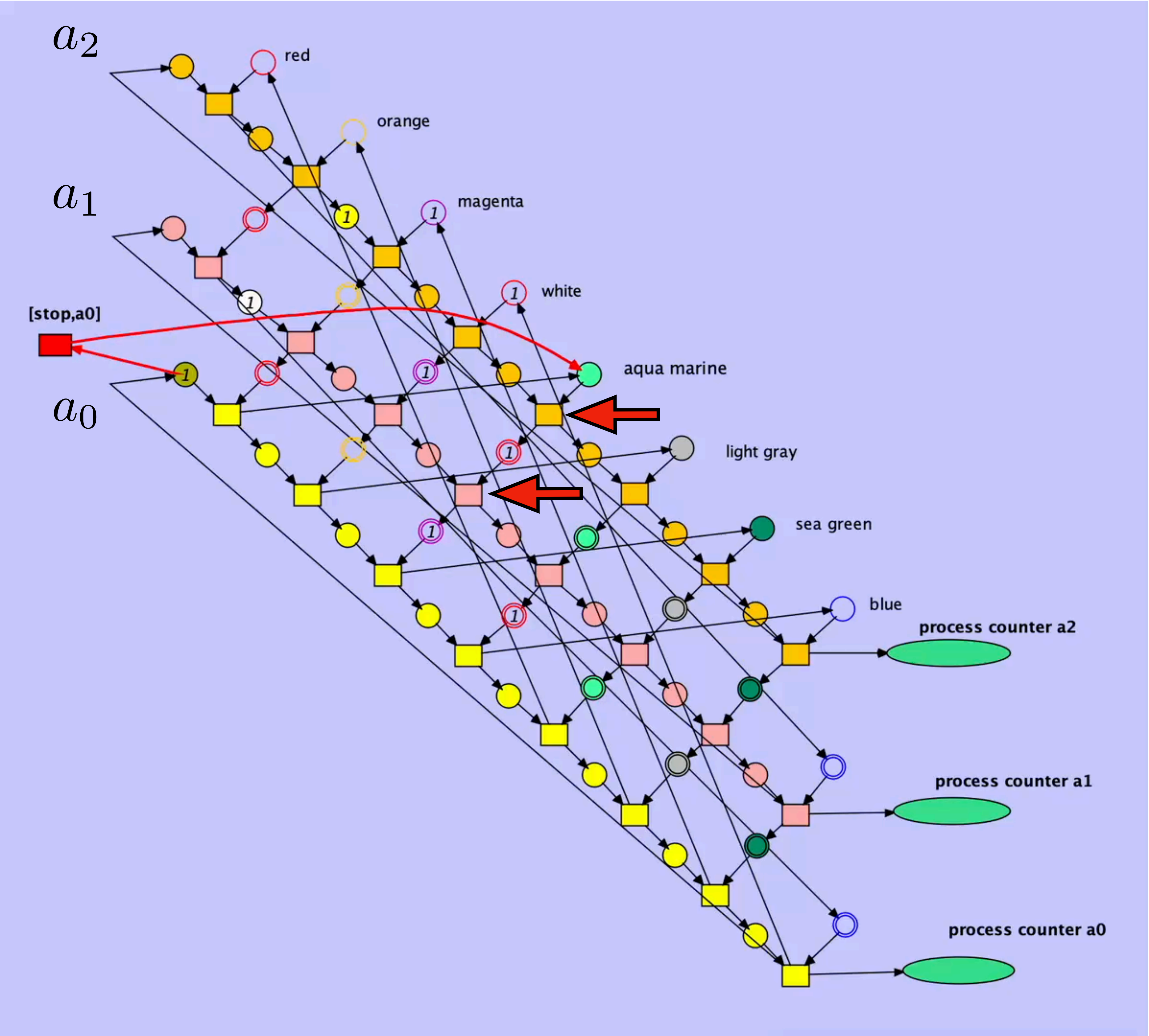
$\mathcal{C}_{bf}(2,3,4,6)$

A deadlock occurs !

work around of the problem :

$p \leq \alpha + \beta$

8 2 + 3



Theorem:

The backward folding is safe and live if $\alpha + \beta \leq p + 1$.

to avoid a deadlock, when a_0 stopps

$$p < \alpha + \beta$$

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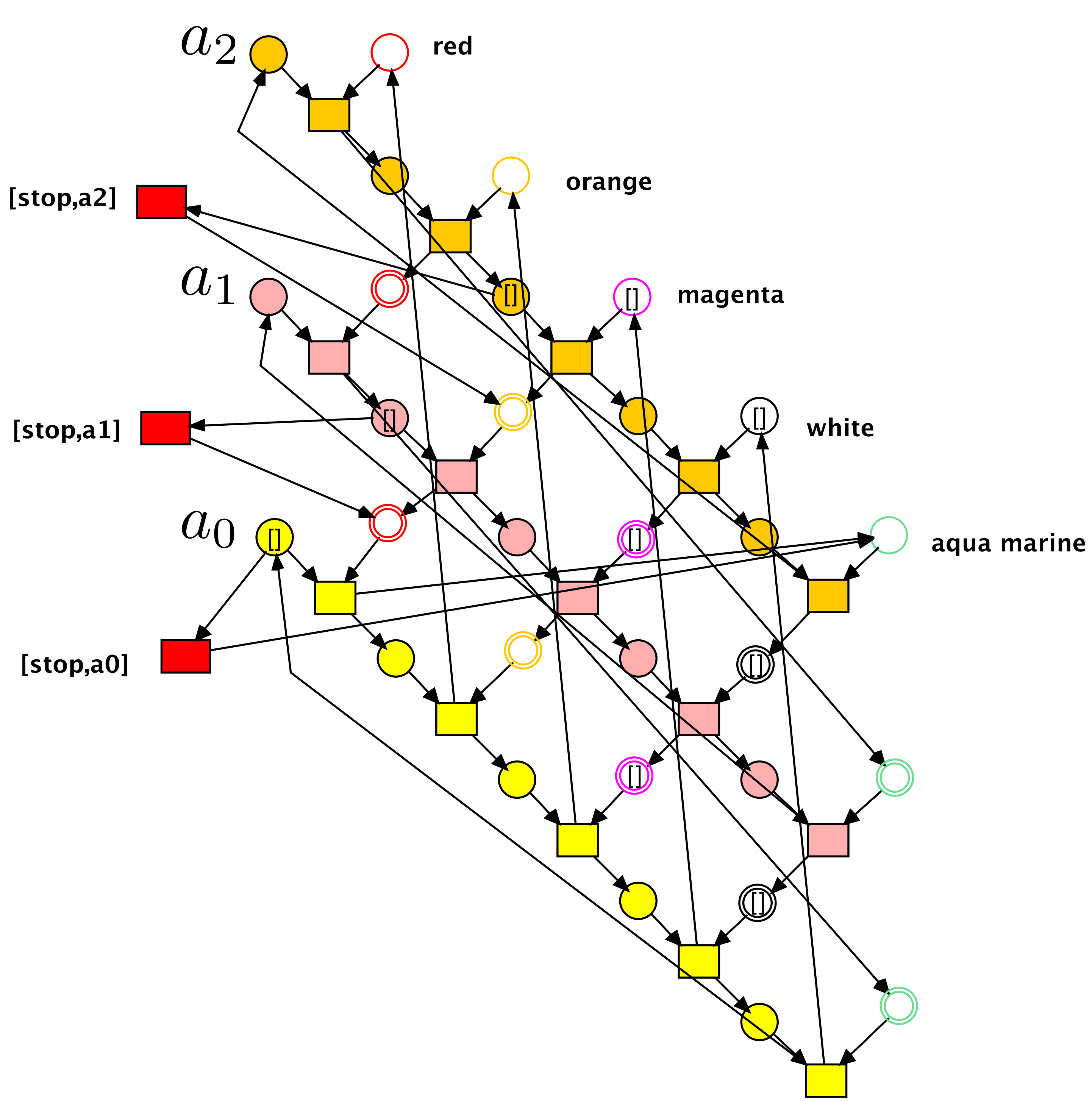
to avoid a deadlock, when a_0 stopps

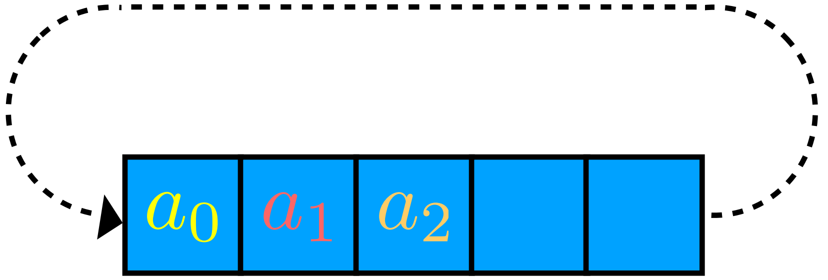
$$p < \alpha + \beta$$

the canonical form of regular cycloid

$$p = \alpha + \beta$$



the canonical form of regular cycloids



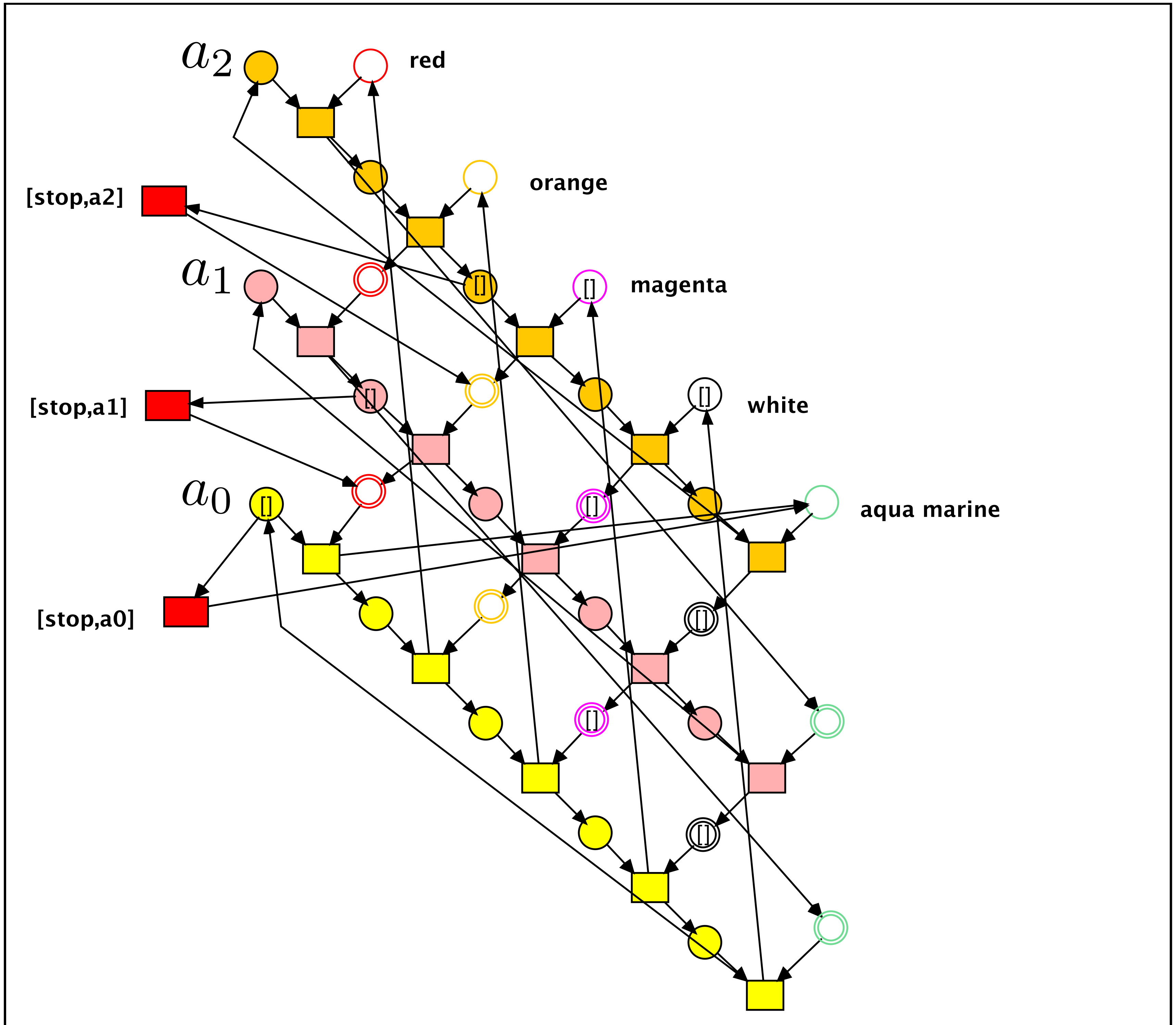
$$\mathcal{C}(\alpha, \beta, \gamma, \delta)$$

$$= \mathcal{C}(2, 3, 3, 3)$$

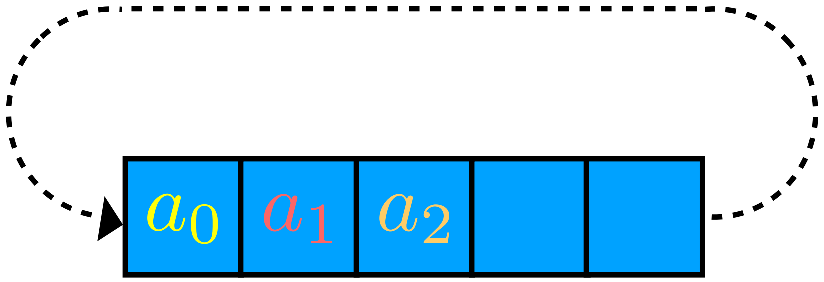
number of gaps

number of processes

process length $= p = \alpha + \beta = 5$



the canonical form of regular cycloids



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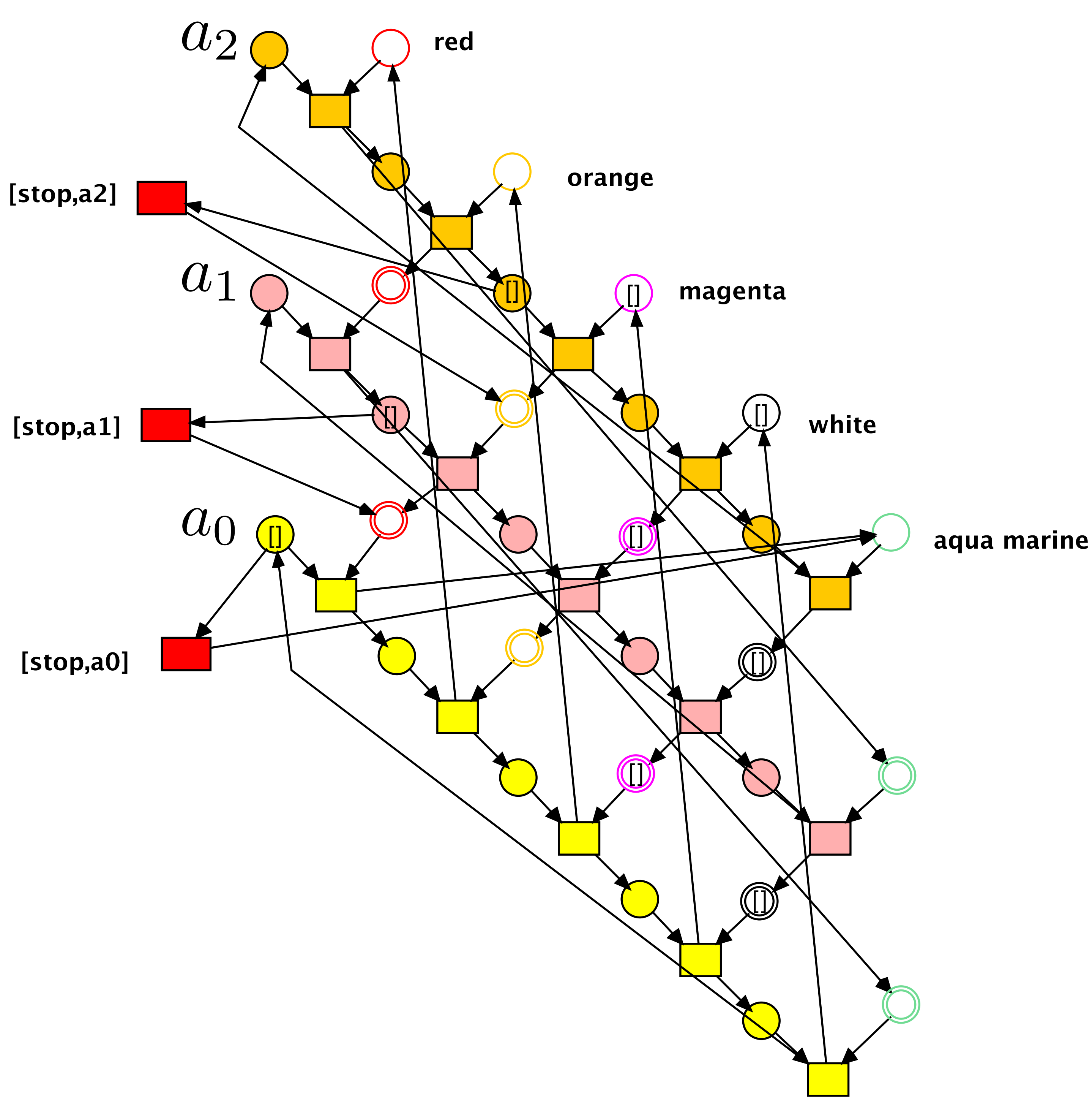
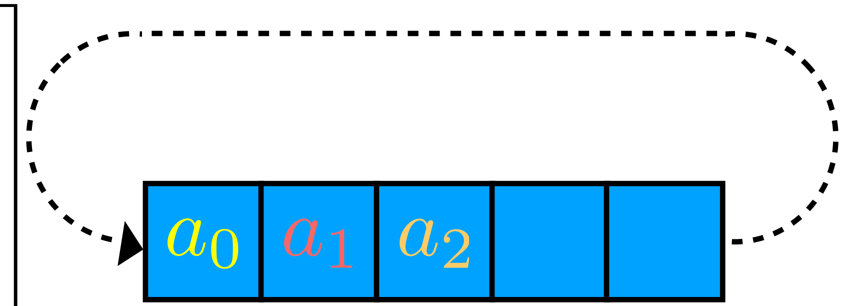


the canonical form of regular cycloids



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$$= \mathcal{C}(2, 3, 3, 3)$$

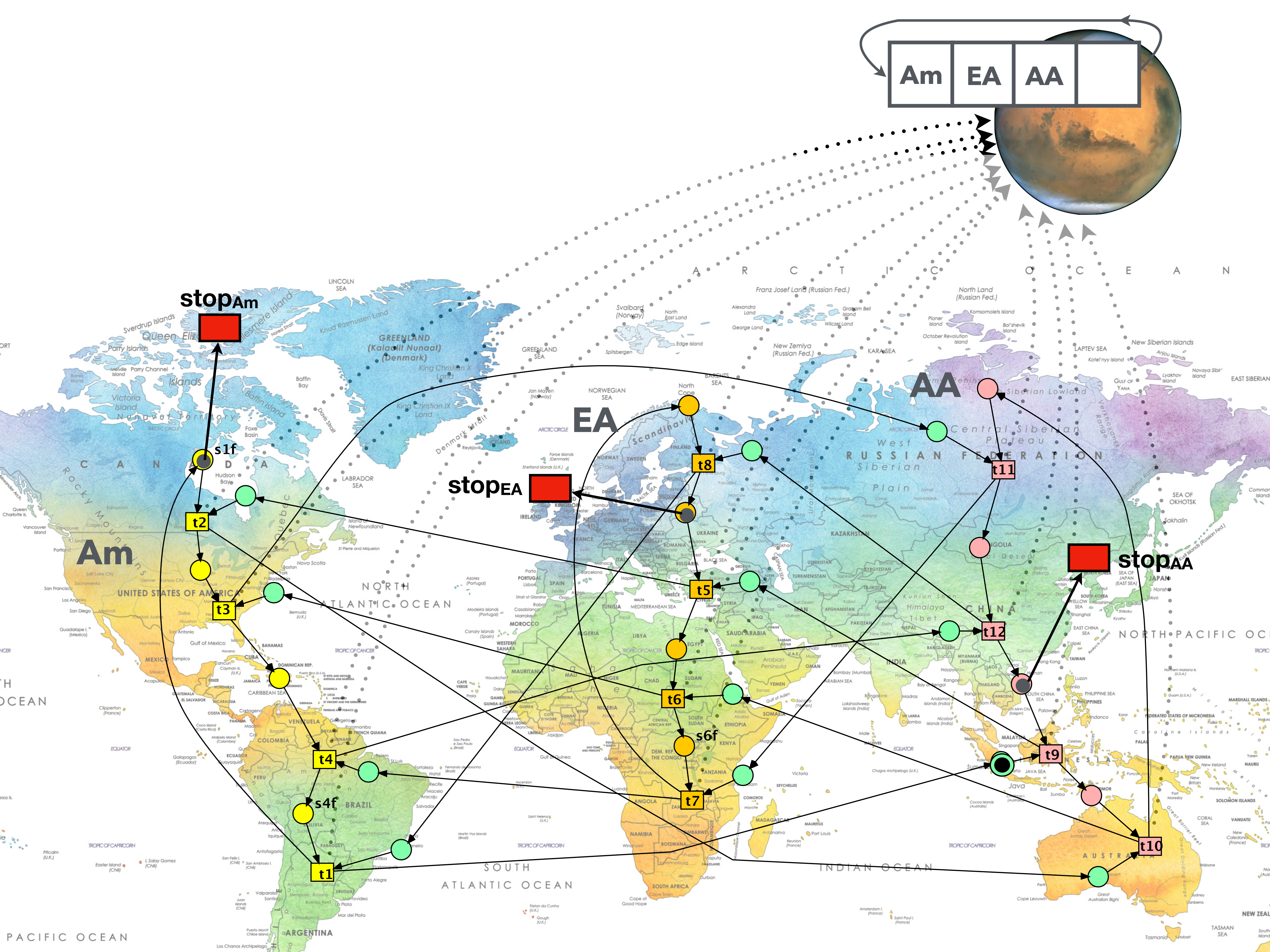
number of gaps

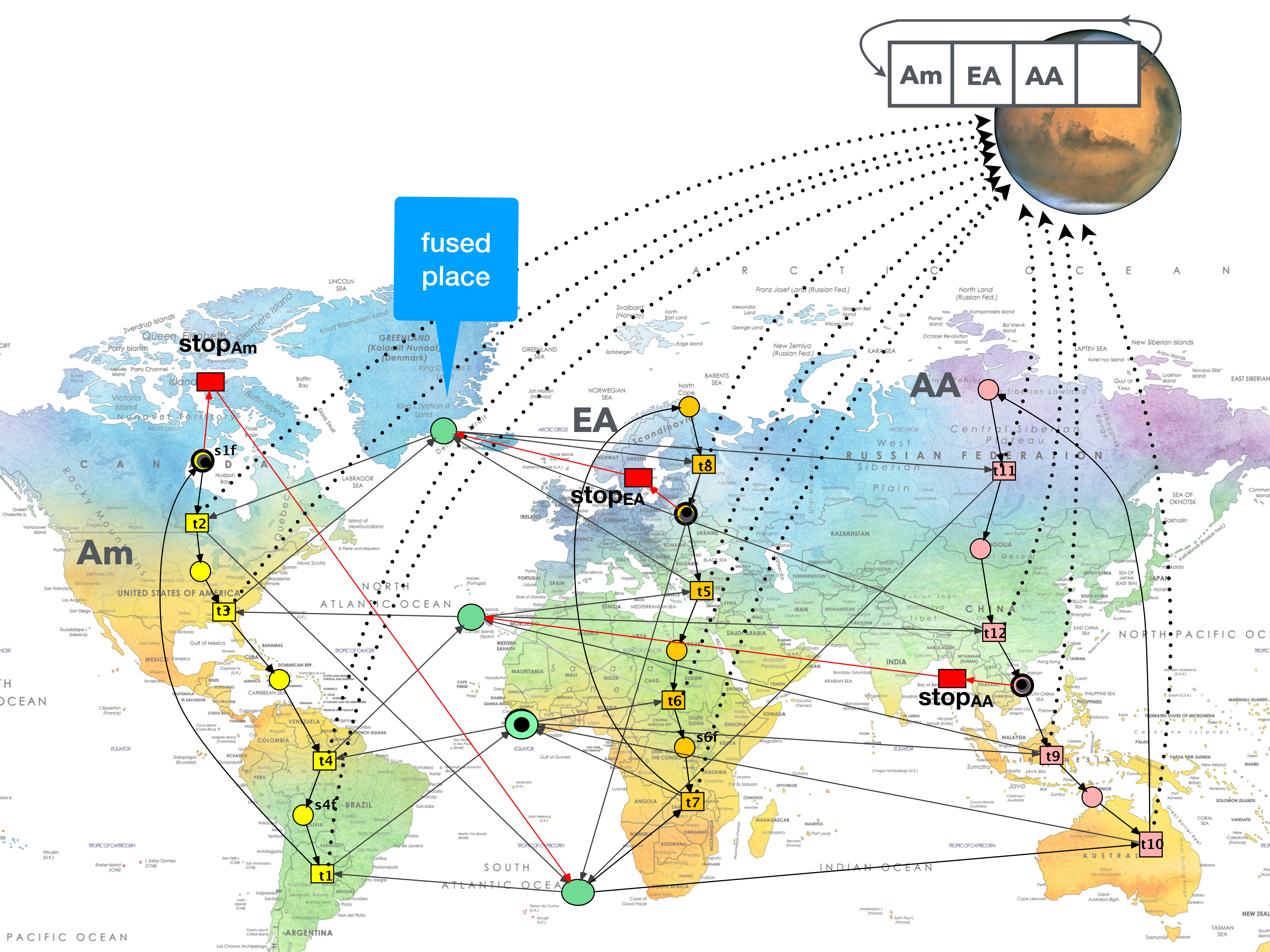
number of processes

process length $= p = \alpha + \beta = 5$

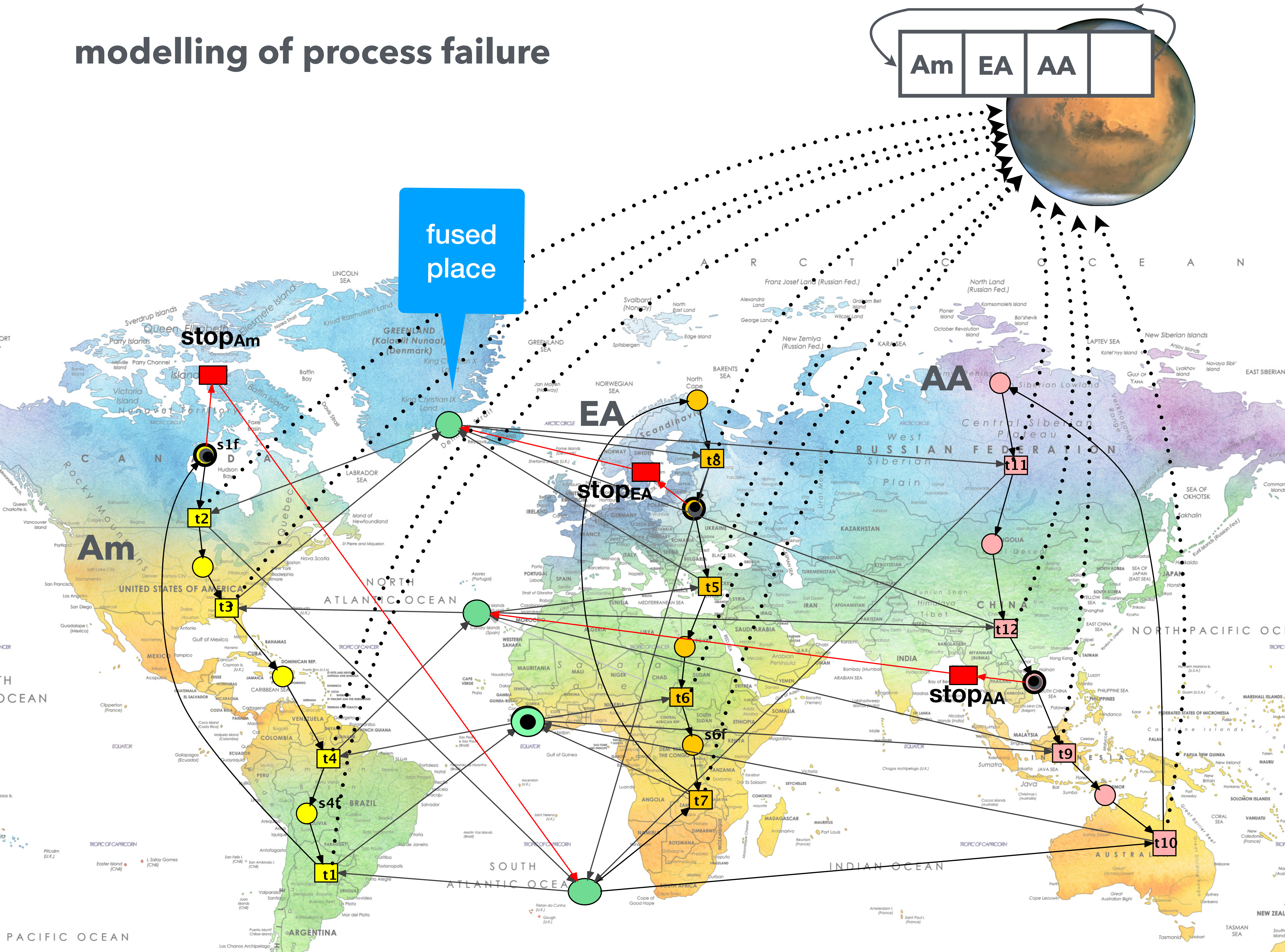
system from
our introduction

$$\mathcal{C}_{bf}(1, 3, 3, 3)$$


Am
EA
AA
stopAm
stopEA
stopAA
s1f
s4f
s6f
t1
t2
t3
t4
t5
t6
t7
t8
t9
t10
t11
t12


Am
EA
AA
fused
place
stopAm
stopEA
stopAA
s1f
s4f
s6f
t1
t2
t3
t4
t5
t6
t7
t8
t9
t10
t11
t12
Am
EA
AA

modelling of process failure



Why using cycloids ?

canonical

$$\mathcal{C}_{bf}(2,3,3,3) \xrightarrow{a_0} \mathcal{C}_{bf}(3,2,2,2) \qquad \mathcal{C}_{bf}(2,3,3,3) \xrightarrow{a_1,\, a_2} \mathcal{C}_{bf}(3,2,2,2)$$

regular,
not canonical

$$\mathcal{C}_{bf}(2,3,4,6) \xrightarrow{a_1,\, a_2} \mathcal{C}_{bf}(3,2,5,2) \qquad \mathcal{C}_{bf}(2,3,4,6) \xrightarrow{a_0} \mathcal{C}_{bf}(6,2,2,2)$$

The resulting structures are known (to some extend)

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$$\mathcal{C}(5, 2^{10000}, 2^{10000}, 2^{10000})$$

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$$\mathcal{C}(5, 2^{10000}, 2^{10000}, 2^{10000})$$

canonical

$\mathcal{C}(2, 3, 3, 3)$

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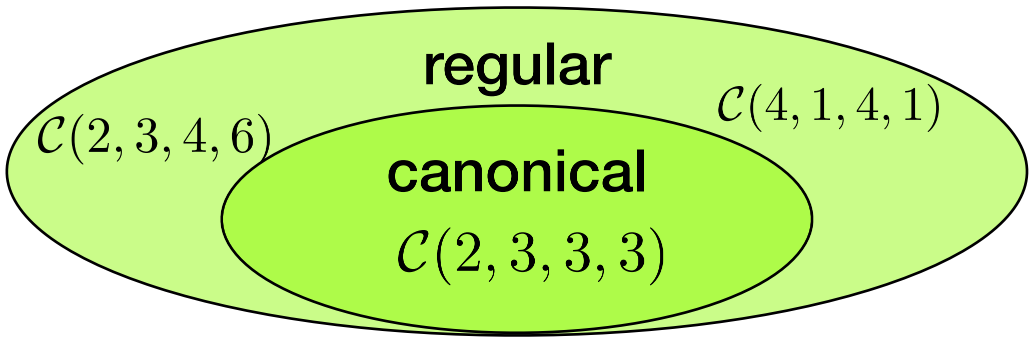

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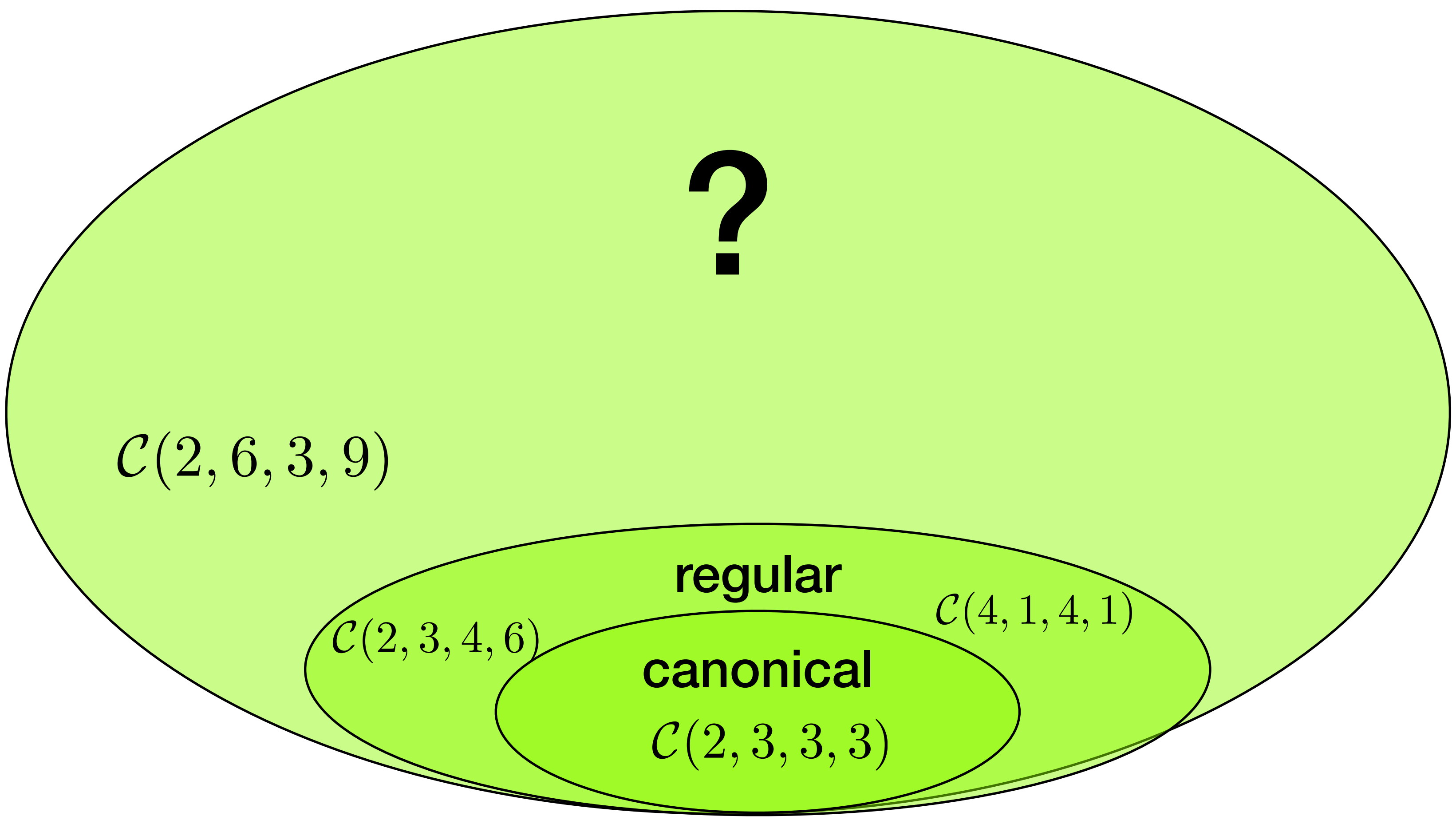
$$\mathcal{C}(5, 2^{10000}, 2^{10000}, 2^{10000})$$



from the theorie on cycloids

$\mathcal{C}(\alpha, \beta, \gamma.\delta)$ $\mathcal{C}(2, 6, 3, 9)$ not regular !

number of transitions: $A = \alpha \cdot \delta + \beta \cdot \gamma$	(by Petri)	36
cooperating processes: "regular" cycloid, if $\beta \vert \delta$		not
number of processes: $gcd(\beta, \delta)$		3
length of minimal cycles	see cycloids webside	8
length of processes: $p = \frac{A}{gcd(\beta,\delta)}$		12
number of tokens in a process: $\frac{\beta}{gcd(\beta, \delta)}$		2

a formula for the synchronic distance

linear decision of cycloid isomorphism by reduction

linear algebra of cycloids

… and much more

from the theorie on cycloids

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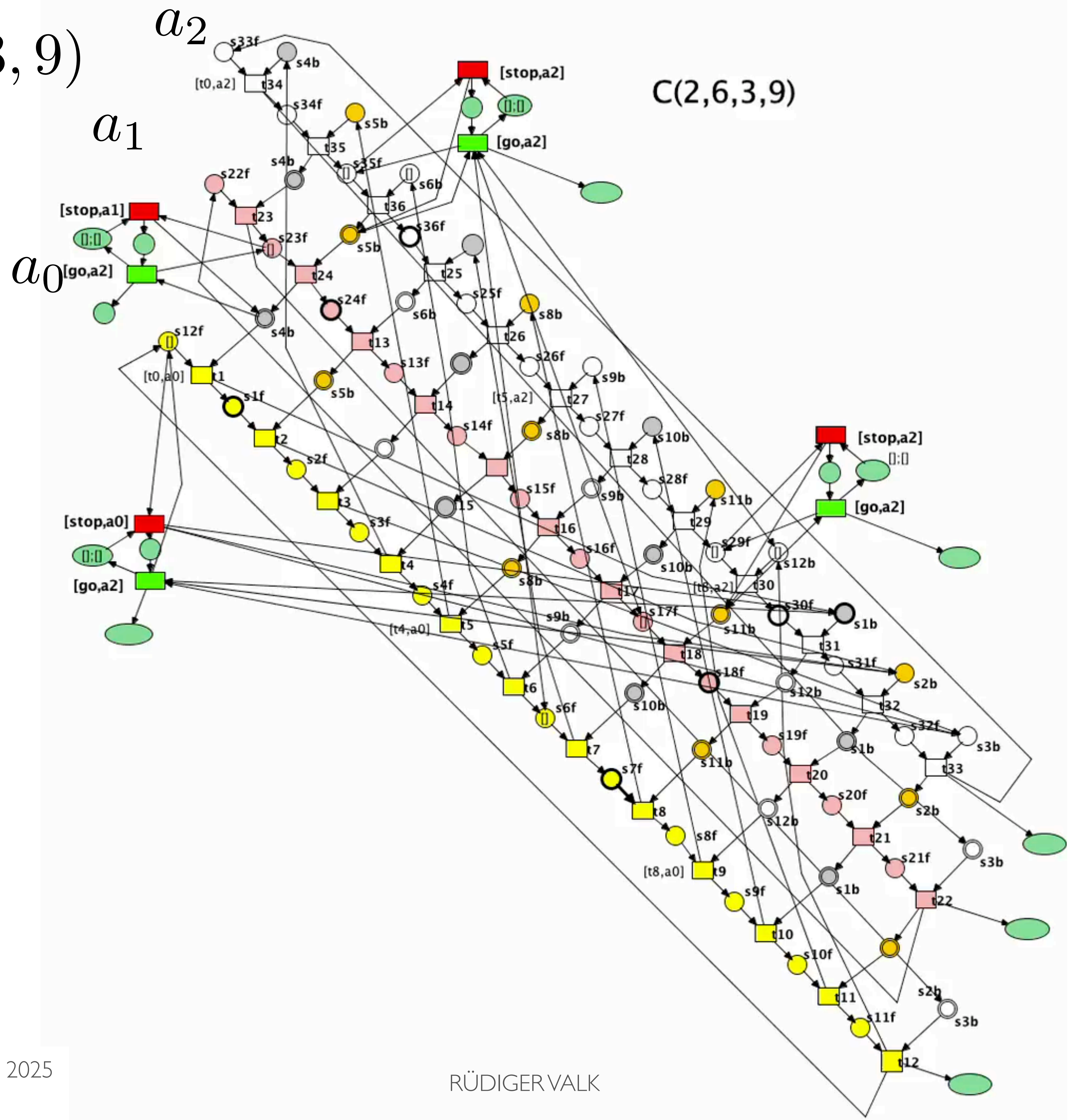
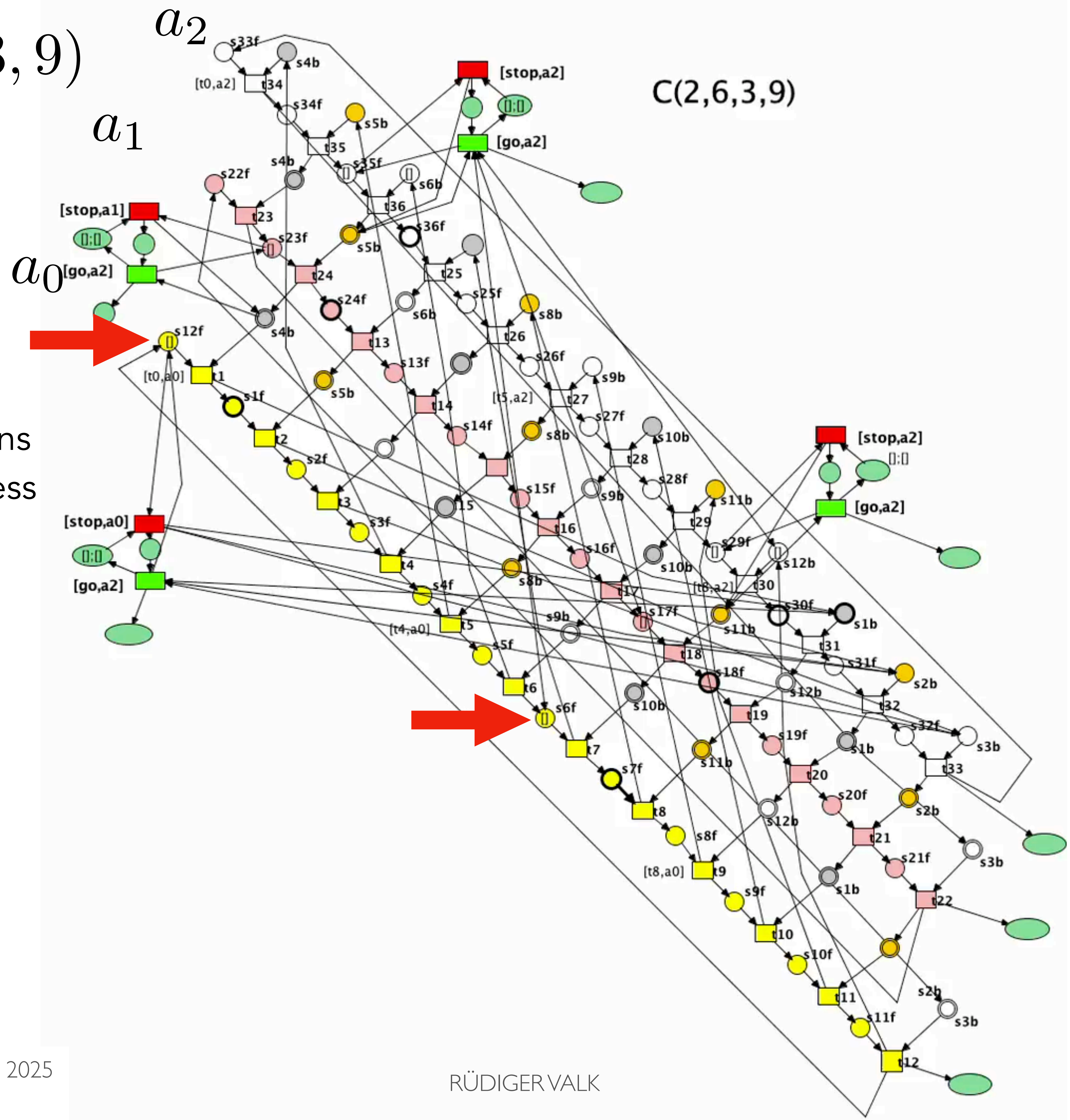
$\mathcal{C}(2,6,3,9)$

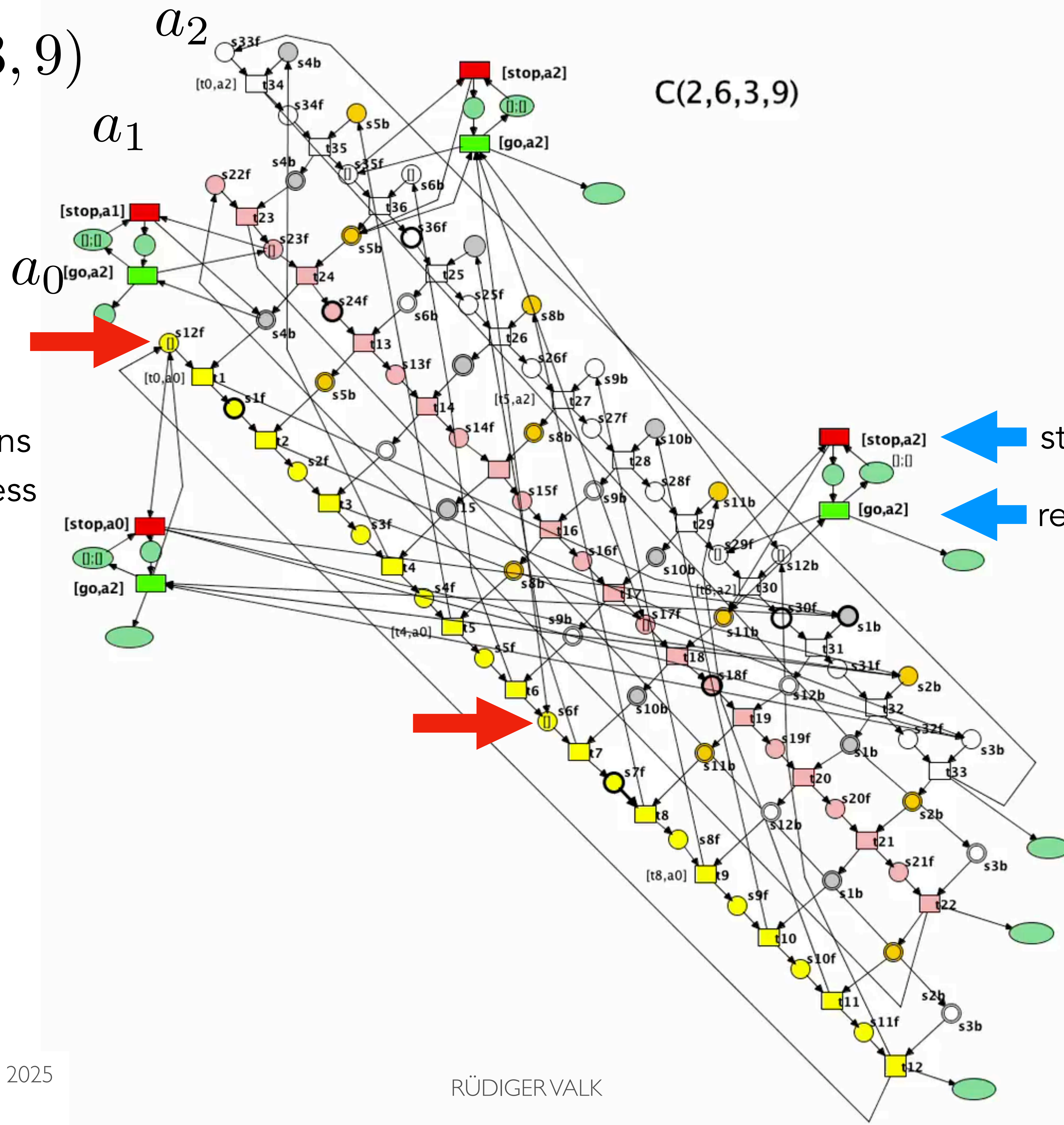


$\mathcal{C}(2,6,3,9)$



two tokens
per process

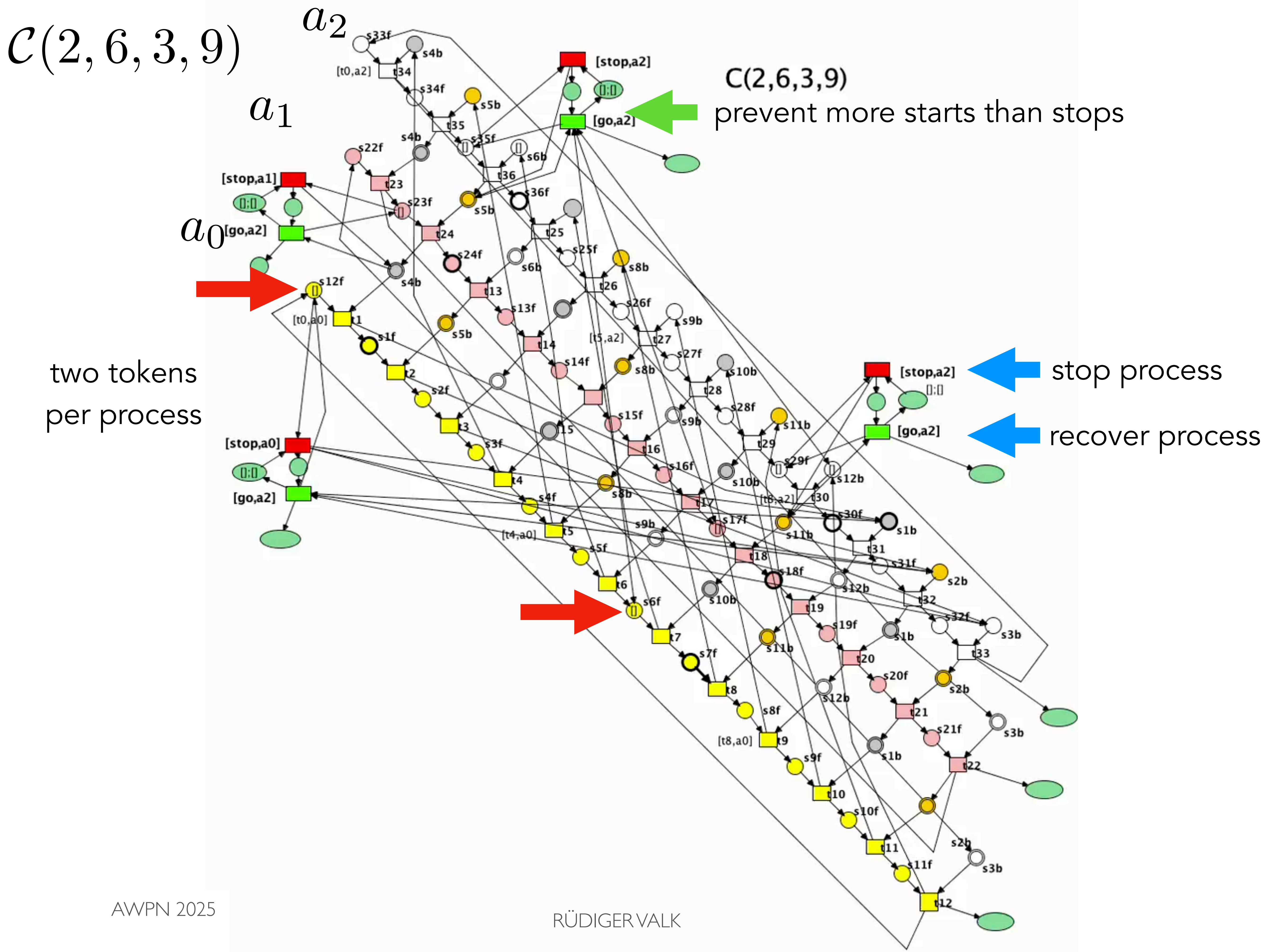
$\mathcal{C}(2,6,3,9)$



two tokens
per process

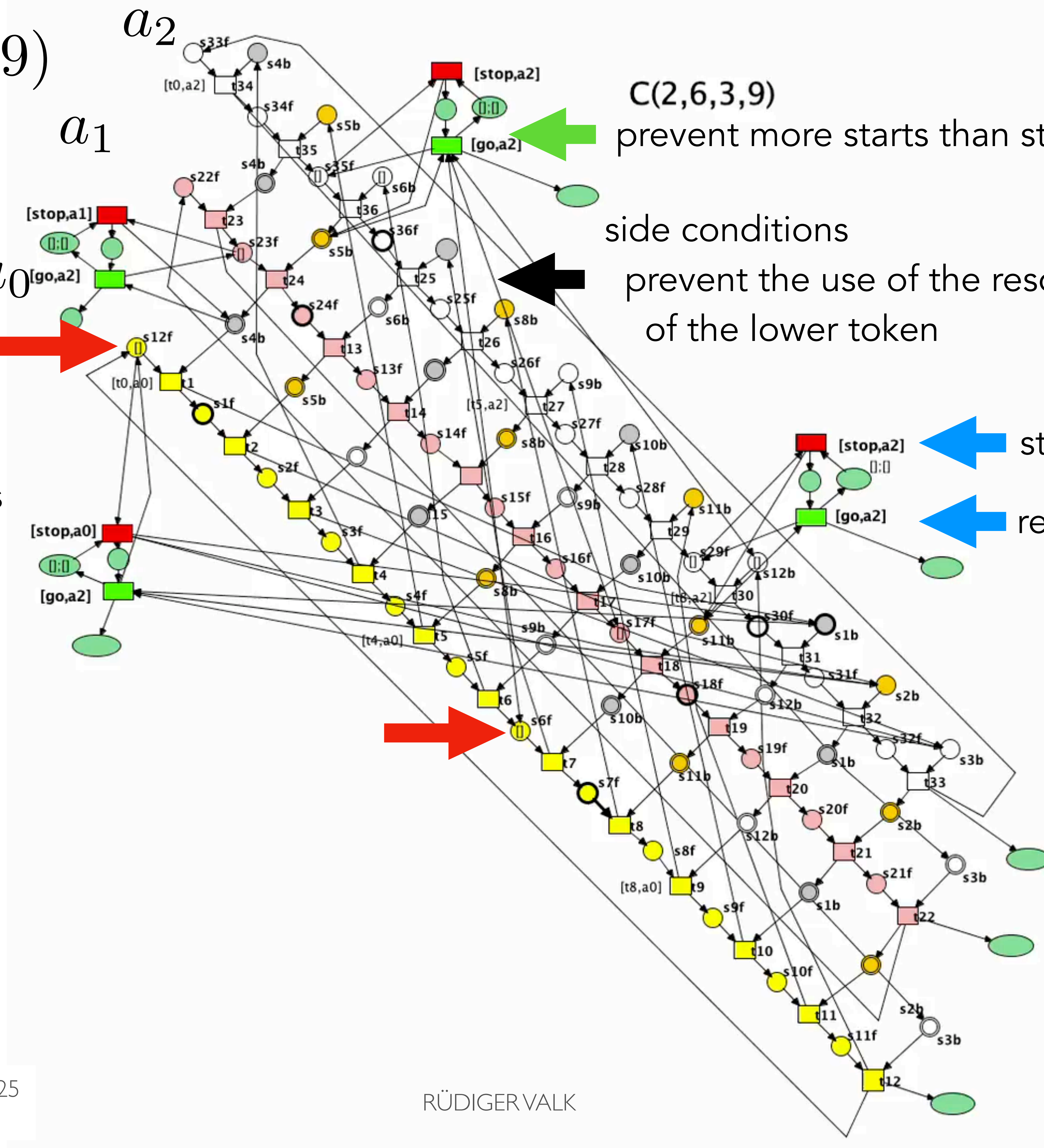
stop process

recover process



$\mathcal{C}(2,6,3,9)$
a_2
a_1
a_0
C(2,6,3,9)
prevent more starts than stops
two tokens
per process
stop process
recover process

$\mathcal{C}(2,6,3,9)$



a_2

a_1

a_0

C(2,6,3,9)

prevent more starts than stops

side conditions

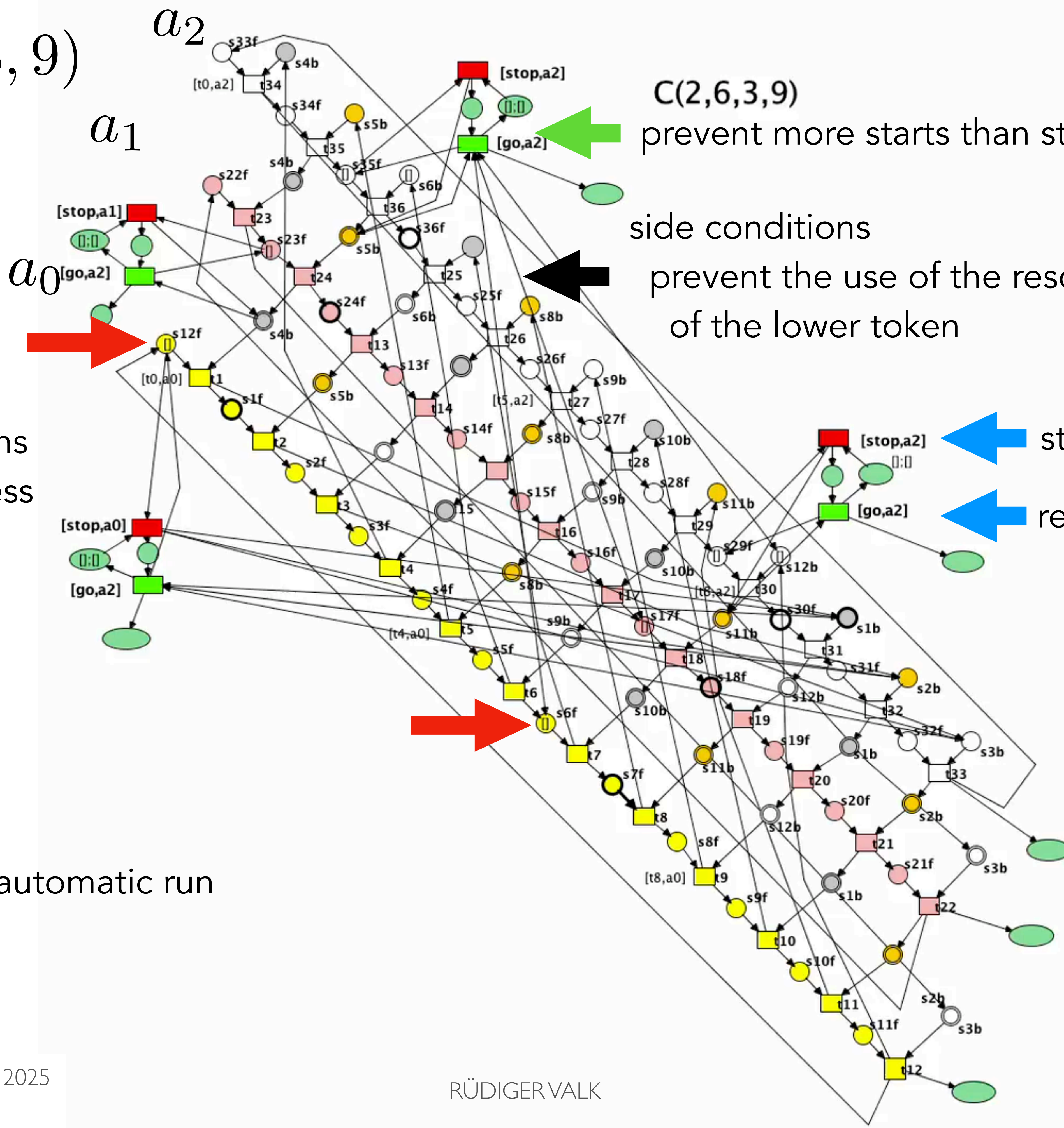
prevent the use of the resources of the lower token

two tokens per process

stop process

recover process

$\mathcal{C}(2,6,3,9)$



C(2,6,3,9)

prevent more starts than stops

side conditions

prevent the use of the resources of the lower token

two tokens per process

stop process

recover process

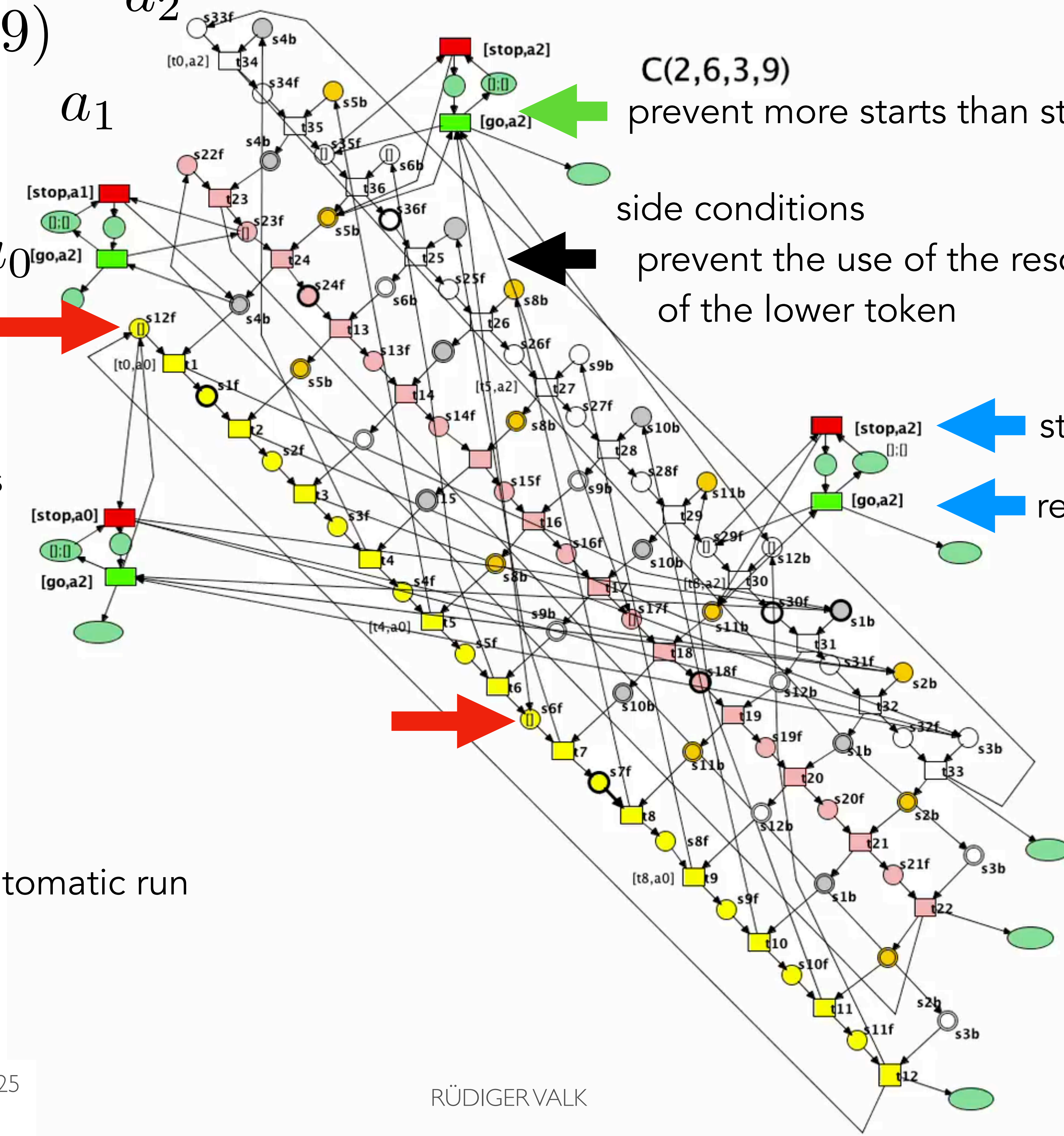
start automatic run

$\mathcal{C}(2,6,3,9)$

a_2

a_1

a_0



C(2,6,3,9)

prevent more starts than stops

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prevent the use of the resources of the lower token

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A lot of information of cycloids can be found on the web side

https://cycloids.de/

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including the slides of this talk

pdf-doc and movie

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Videos and Presentations on Cycloids

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References to Cycloids

* * *

Find the equivalent point within the Fundamental Parallelogram

* * *

Algorithms on Cycloids

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Thank you for your attention !

RÜDIGER VALK

UNIVERSITY OF HAMBURG

AWPN 2025

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